Learning and Evaluation Situation

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SSC1: Solve a Situational Problem

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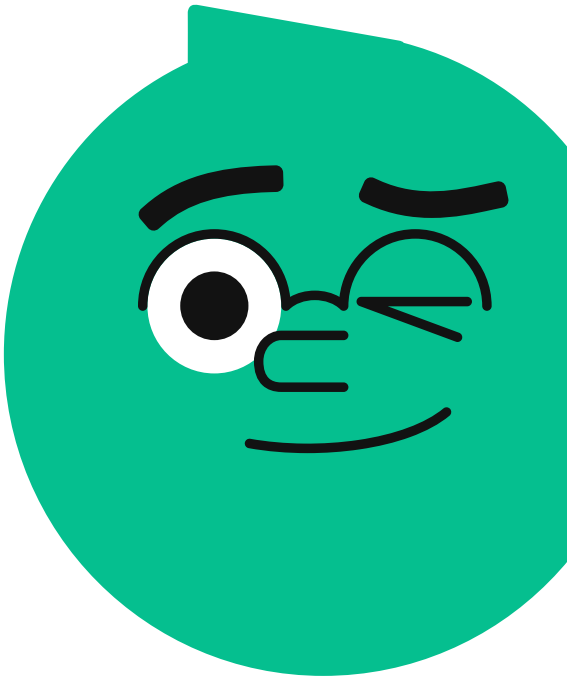
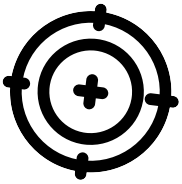


Mathematics

Secondary IV – SN

The Forges of Farador

Teacher Booklet



Activity Duration:

Three 75-minute periods

Materials:

For the teacher: one *Teacher Booklet*
For each student: one *Student booklet - Instructions* and one *Student booklet - Answers*

Procedure

This situational problem is appropriate for the end of the school year, as it contains most of the concepts covered in the Secondary IV Science Option. However, if you decide to give it to your students earlier in the school year, you can give them certain answers. That way, they can work through the steps that relate to concepts already covered in class without getting stuck on the steps they are not yet able to complete. You can also take this approach to shorten the total duration of the situational problem.

For example, we have decided to shorten the duration of the **bow** section by providing the price per rune ($10) and per centimetre of wood ($0.50). But if you want your students to solve a system of two equations with two unknowns in a non-geometry context, you could change the sentence, “Each engraved rune costs $10, plus $0.50/cm of wood” to: “A 120 cm bow with 10 runes costs $160, while a 150 cm bow with 5 runes costs $125.”

This situational problem should be done over the course of three periods. During the first and second periods, students will work individually to calculate the selling price of each piece of equipment (shield, sword, dagger, staff, bow, and arrows). During the third period, students will work in teams of two or three to compare their results and determine a selling price for each set that respects the requirements set out in the problem.

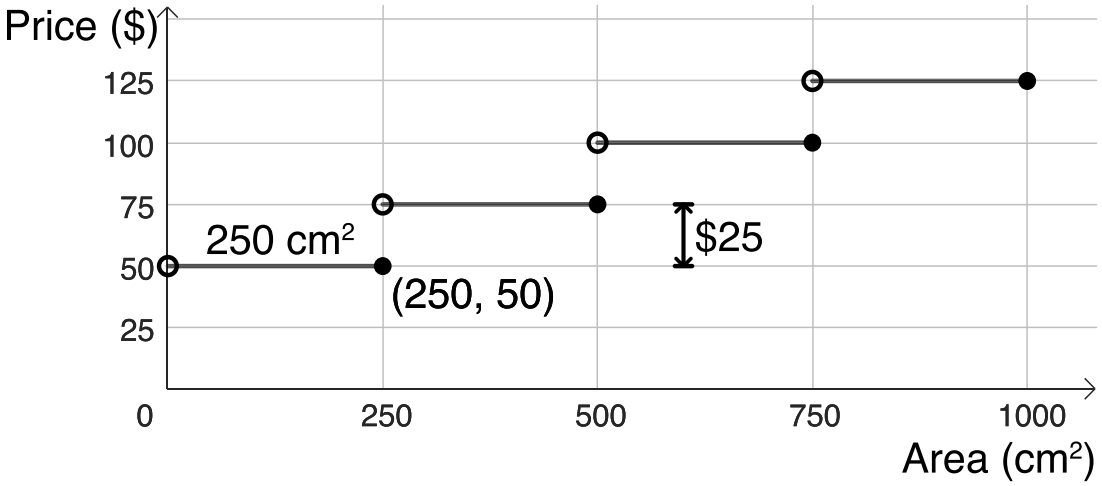
Here is a list of the concepts in this situational problem, as well as the information we suggest you give to your students if you decide to shorten the duration of each section.

Section	Concept	Information to give to the student
Styrofoam (dagger, sword, and staff)	Quadratic polynomial functions	$p(x) = -0.02(x - 100)^2 + 200$ Dagger: $38 Sword: $187.50 Staff: $198
Shield area	Analytic geometry • Perpendicular lines • Distance between two points • Etc.	A_t = 600 cm² = 6 dm²
Shield selling price	Greatest integer function	$f(x) = -25[-0,4(x - 2,5)] + 50$ where f(x): cost ($) and x: area (dm²) f(6) = $100
Area of an arrowhead	Metric relationships in right triangles	A = 44,09 cm² Cout : $7,41
Bow: Find the length of the bowstring, the length of the bow, and the price of the bow	Law of sines	Length of bowstring: 153.08 cm Length of bow: 183.7 cm Price of bow with 8 engravings: $171.85

Answer key

Shield

Formula for the shield price

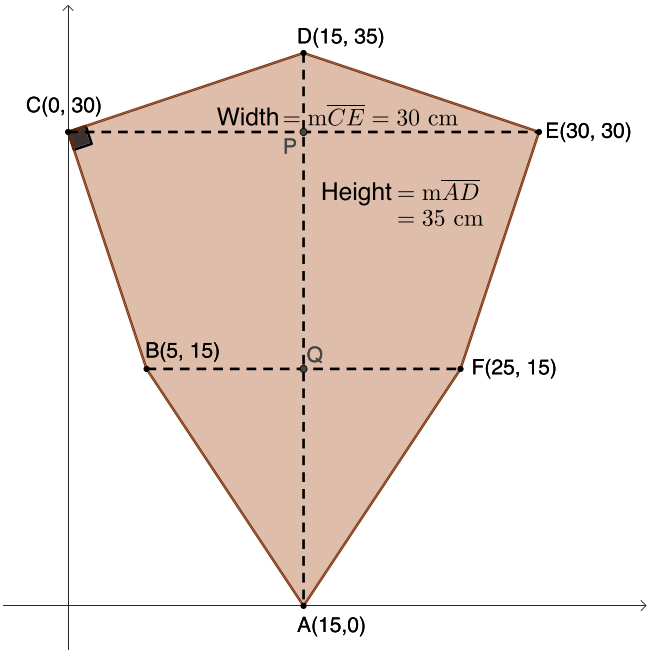


$$p(x) = -25\left[-\frac{1}{250}(x - 250)\right] + 50$$

where

x: Shield area (cm²)
p(x): Shield price ($)

Area of the shield to be sold



Justifications expected of the students

- Find the coordinates of the various points
  - The ordinate of A is 0, because A is on the x-axis.
  - The abscissa of C is 0, because C is on the y-axis.
  - The ordinate of D is 35, because the height of the shield is 35 cm and the lower vertex (A) lies on the x-axis.
  - The abscissa of vertices A and D is 15, because $\overline{AD}$ is an axis of symmetry of the shield. Therefore,

$$x_A = x_D = \frac{x_E - x_C}{2} = \frac{30 - 0}{2} = 15$$

- The ordinate of point C is 30, because C is symmetrical with E. Therefore, $x_C = x_E = 30$.
- The formula for segment $\overline{CD}$ is: $y = \frac{1}{3}x + 30$
- The slope of segment $\overline{CB}$ is 3, because $\overline{CB}$ and $\overline{CD}$ are perpendicular.
- The formula for segment $\overline{CB}$ segment is: $y = -3x + 30$
- The coordinates of B are: (5, 15).
- The coordinates of F are: (25, 15) because F is symmetrical to B in relation to $\overline{AD}$.

- Calculate the shield area
  Note: Students can divide their shields in several different ways.
  Here is one possibility.
  - Triangle CDE
    - Base of triangle CDE = dist(C,E) = 30 cm
    - Height of triangle CDE = $y_D - y_C$ = 35 - 30 = 5 cm

$$A_{\triangle CDE} = \frac{b \times h}{2} = \frac{30 \times 5}{2} = 75 \text{ cm}^2$$

  - Trapezoid CEFB
    - Large base = dist(C,E) = 30 cm
    - Small base = dist(B,F) = 25 - 5 = 20 cm
    - Height = $y_C - y_B$ = 30 - 15 = 15 cm

$$A_{CEFB} = \frac{(B + b) \times h}{2} = \frac{(30 + 20) \times 15}{2} = 375 \text{ cm}^2$$

  - Triangle BFA
    - Base of triangle BF = dist(B,F) = 20 cm
    - Height of triangle BFA = $y_B - y_A$ = 15 - 0 = 15 cm

$$A_{\triangle BFA} = \frac{b \times h}{2} = \frac{20 \times 15}{2} = 150 \text{ cm}^2$$

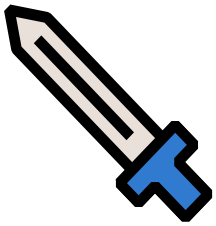
  - Total area
    - TA = 75 + 375 + 150 = 600 cm²

Therefore, the price of this shield is calculated by replacing x with 600 in the formula found earlier, or by using the mock-up of the shield provided.

$$p(600) = -25\left[-\frac{1}{250}(600 - 250)\right] + 50 = \$100$$

Sword, Dagger, Wizard's Staff

- This is a quadratic polynomial function.
- We are given point (10, 38), because according to the domain, the shortest sword measures 10 cm and sells for $38.
- We are also given the vertex (100, 200).
- The formula is: $p(x) = -0.02(x - 100)^2 + 200$



- The 10 cm dagger (smallest possible piece) costs $38.
- The 75 cm sword costs $187.50.
- The 110 cm staff (largest possible piece) costs $198.

Arrows

- $\mathrm{m}\overline{AB}^{\,2} = \mathrm{m}\overline{AE} \times \mathrm{m}\overline{AD}$

  because in a right triangle, each leg is the mean proportional between its projection onto the hypotenuse and the entire hypotenuse.
- The result is a quadratic polynomial function. The solution to this equation is the measurement of segment $\overline{AE}$.

$$3^2 = x(x + 14.4)$$

$$0 = x^2 + 14.4 - 9$$

$$x = \frac{-14.4 \pm \sqrt{14.4^2 - 4(1)(-9)}}{2(1)}$$

$$x_1 = 0.6 \text{ cm} \qquad \underbrace{x_2 = -15 \text{ cm}}_{\text{rejected}}$$

⇒ Segment $\overline{AE}$ measures 0.6 cm.
Note: Students should explain why they reject -15 cm.

- $\mathrm{m}\overline{BE}^{\,2} = \mathrm{m}\overline{AE} \times \mathrm{m}\overline{ED}$

  because in a right triangle, the height from the right angle is the proportional mean between the two segments it forms on the hypotenuse.
- Segment $\overline{BE}$ (i.e., the height of triangle ABD) measures approx. 2.94 cm.

- $$\begin{aligned} \mathrm{A}_{\triangle ABD} &= \frac{\mathrm{m}\overline{AD} \times \mathrm{m}\overline{BE}}{2} \\ &= \frac{(14.4 + 0.6) \times 2.94}{2} \\ &22.05 \text{ cm}^2 \end{aligned}$$

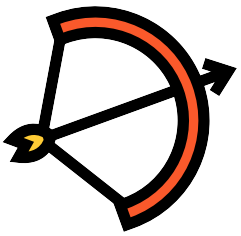
- Area of the arrowhead = 2 × 22.05 = 44.1 cm²
- Cost for 1 arrow: f(44.1) = 0.1(44.1) + 3 = $7.41
- Cost for 15 arrows: $111.15

Bow

- $\mathrm{m}\overline{BD} = 70 \text{ cm}$, because segment $\overline{BD}$ corresponds to the length of the arrow shaft
- Law of sines:

$$\begin{aligned} \frac{\mathrm{m}\overline{BD}}{\sin C} &= \frac{\mathrm{m}\overline{CD}}{\sin B} \\ \frac{70}{\sin C} &= \frac{80}{\sin(66^\circ)} \\ \mathrm{m}\angle C &= \sin^{-1}\left(\frac{70\sin(66^\circ)}{80}\right) \\ \mathrm{m}\angle C &\approx 53.07^\circ \end{aligned}$$

- $$\begin{aligned} \mathrm{m}\angle BDC &= 180^\circ - 66^\circ - 53.07^\circ \\ &= 60.93^\circ \end{aligned}$$



- $$\begin{aligned} \frac{\mathrm{m}\overline{BC}}{\sin D} &= \frac{\mathrm{m}\overline{CD}}{\sin B} \\ \frac{\mathrm{m}\overline{BC}}{\sin(60.93^\circ)} &= \frac{80}{\sin(66^\circ)} \\ \mathrm{m}\overline{BC} &= \frac{80\sin(60.93^\circ)}{\sin(66^\circ)} \\ \mathrm{m}\overline{BC} &\approx 76.54 \text{ cm} \end{aligned}$$

- Length of bowstring = 2 × 76.54 = 153.08 cm
- Length of bow = 120% of 153.08 = 183.7 cm
- Cost of the bow = 183.7 cm × $0.50/cm + 8 engravings × $10/engraving = $171.85

Sets and Final Answers

Equipment	Price per item (no discount)
Shield	$100
Sword	$187,50
Dagger	$38
Staff	$198
Arrow	$7,41
Bow	$171,85

Set	Price for set (before discount)	Price for set (with discount)
Gardakan (Shield and sword)	$287,50	a) 12 % discount ⇒ $253 b) 13 % discount ⇒ $250,13 c) 14 % discount ⇒ $247,25 **d) 15 % discount ⇒ $244,38*** **e) 16 % discount ⇒ $241,50*** **f) 17 % discount ⇒ $238,63*** **g) 18 % discount ⇒ $235,75***
Mordak (dagger and staff)	$236	$209,85 (11,08% discount)
Boba Fett (bow and 15 arrows)	$283	231,13 $ (18,33% discount)
Total	$806,50	a) $693,98 (a savings of $112,52) b) $691,11 (a savings of $115,40) c) $688,23 (a savings of $118,27) **d) $685,36** **(a savings of $121,15)** **e) $682,48** **(a savings of $124,02)** **f) $679,61** **(a savings of $126,90)** **g) $676,73** **(a savings of $129,77)**

*There are 4 possible solutions. Students don't have to find them all.

- Students may also apply a 15% discount to the Gardakan set.
  - 15 is a whole number.
  - 15% is between 11.08% and 18.33%.
  - The resulting savings is $121.15, which is greater than $120.
- Students may also apply a 16% discount to the Gardakan set.
  - 16 is a whole number.
  - 16% is between 11.08% and 18.33%.
  - The resulting savings is $124.02, which is greater than $120.
- The student may also apply a 17% discount to the Gardakan set.
  - 17 is a whole number.
  - 17% is between 11.08% and 18.33%.
  - The resulting savings is $126.90, which is greater than $120.
- Students may also apply an 18% discount to the Gardakan set.
  - 18 is a whole number.
  - 18% is between 11.08% and 18.33%.
  - The resulting savings is $129.77, which is greater than $120.