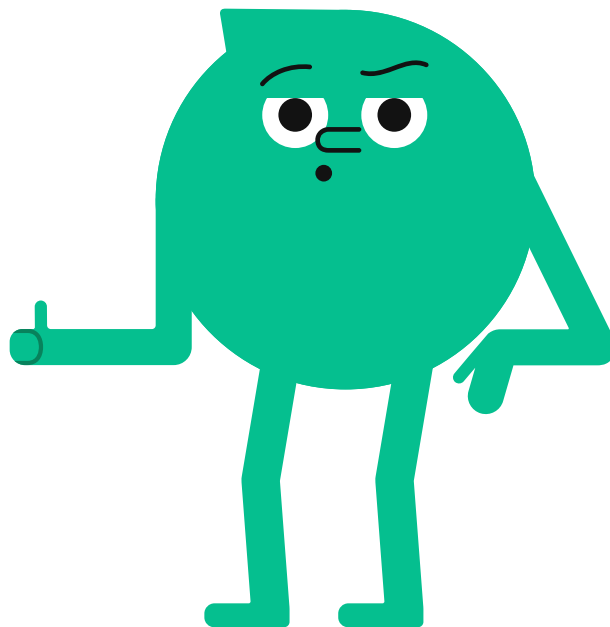


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Mathematics SN Secondary 4

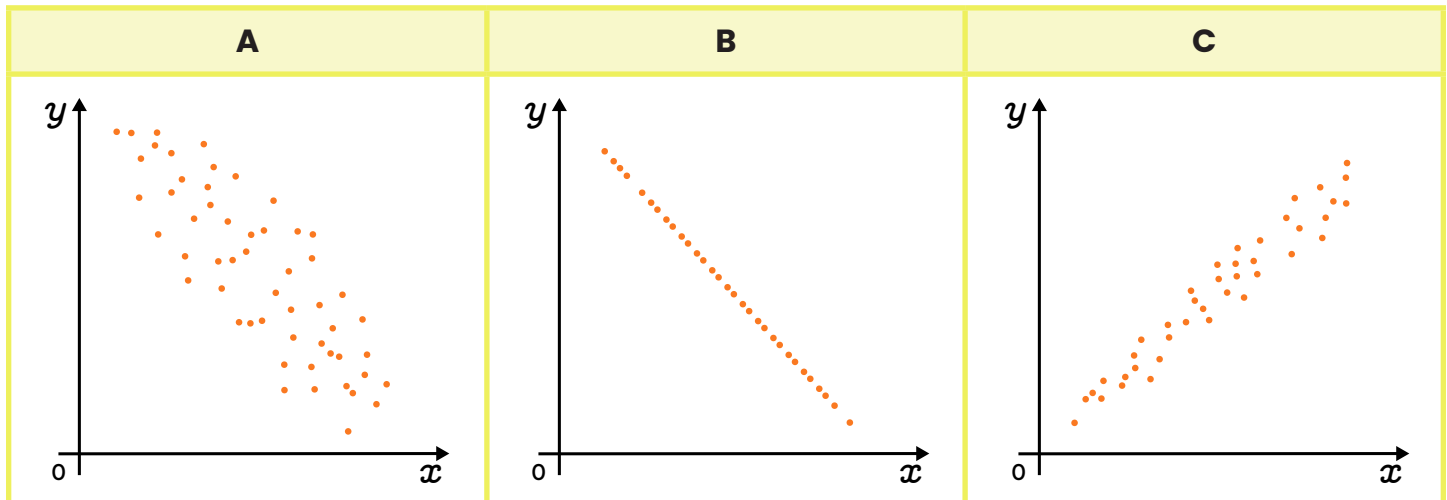


Answer Key

Section A

Question 1

Place the following scatter plots in **descending** order of linear correlation **coefficient**.



- a) $B > C > A$
 - b) $A < C < B$
 - c) $C > A > B$
 - d) $B < A < C$

Answer: c) $C > A > B$

Explanations

Scatter plot **C** is the only graph displaying an **upward** trend. It is therefore the only one with a **positive** correlation coefficient. Because the remaining 2 are negative, C must necessarily have the highest correlation coefficient.

Tip: Because they should be arranged in *descending order*, the correct answer must begin with **C**. Therefore, **by process of elimination**, choices a), b), and d) can be ruled out.

Let's go into more detail. We'll start by estimating the correlation coefficients of the 3 scatter plots.

- Graph **A** shows a **weak negative** correlation. Therefore, $r_A \approx -0.6$.
- Graph **B** shows a **perfect negative** correlation. Therefore, $r_B = -1$.
- Graph **C** shows a **moderate positive** correlation. Therefore, $r_C \approx +0.8$.

Thus, in *descending order*, we get $+0.8 > -0.6 > -1$, i.e., answer **c) $C > A > B$** . Answer **d) $B < A < C$** is incorrect because it appears in *ascending* rather than *descending order*.

If we had been asked to list the scatter plots in order of correlation **strength**, we would have ignored the + and - signs.

- The scatter plot with the weakest correlation is graph A.
- Scatter plot C has a stronger correlation than scatter plot A.
- Scatter plot B has an even stronger correlation because it is perfect.

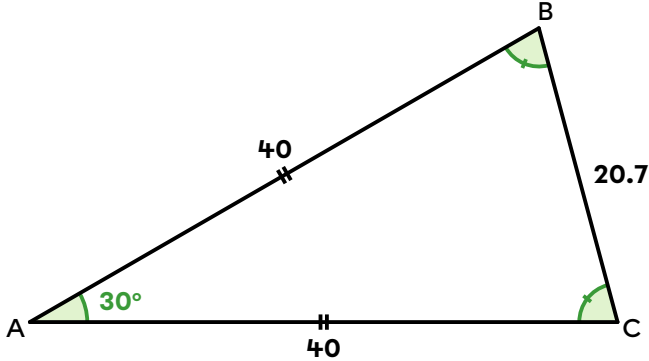
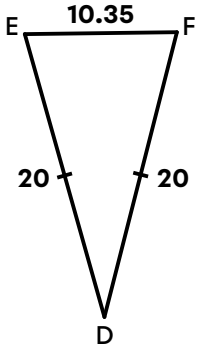
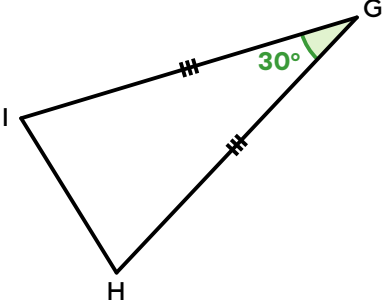
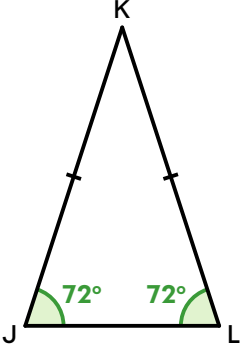
Thus, from weakest to strongest correlation (regardless of signs), the order is **b) $A < C < B$** . From strongest to weakest correlation, the order is reversed, i.e., option **a) $B > C > A$** .

Keep in mind: During the exam, be sure to pay attention to the order of the information (ascending or descending) and what must be put in order, e.g., the correlation **coefficients** (take the plus and minus signs into account) or correlation **strength** (ignore the signs).

Question 2

Which of these doesn't belong?

All but one of the following triangles are similar. Which one?

A	B
	
C	D
	

Answer: d) triangle JKL

- Triangle ABC is similar to triangle DEF by **SSS**: Triangles are similar if their corresponding sides are proportional.

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$$

$$\frac{40}{20} = \frac{40}{20} = \frac{20.7}{10.35}$$

$$2 = 2 = 2$$

- Triangle ABC is similar to triangle GHI by **SAS**: Triangles are similar if they have 1 pair of congruent angles between 2 pairs of proportional sides.

$$\circ m\angle BAC = m\angle HGI = 30^\circ$$

$$\circ \frac{m\overline{AB}}{m\overline{GH}} = \frac{m\overline{AC}}{m\overline{GI}}$$

$$\frac{40}{m\overline{GH}} = \frac{40}{m\overline{GI}}$$

This equality is true because segments GH and GI are congruent.

Note: In these 2 triangles, the congruent angles are indeed located between the proportional sides.

Since triangle ABC is similar to triangle DEF **and** triangle GHI, we can assert, via the transitive property, that triangles DEF and GHI are similar to each other. In other words, we've already identified the 3 similar triangles. By process of elimination, the remaining triangle, KJL, doesn't belong. Here's why.

- Triangles ABC and KJL appear similar by AA (triangles are similar if they have 2 pairs of congruent angles), but they are not. Angles B and C are congruent to each other, as are angles J and L, but this doesn't mean that angle C is congruent to angle J. Let's see why.

In triangle ABC, angle A measures 30° . Since the sum of a triangle's interior angles equals 180° , we can calculate the measures of angles B and C.

$$m\angle B = m\angle C = \frac{180^\circ - 30^\circ}{2} = 75^\circ$$

In triangle KJL, angles J and L measure 72° , not 75° .

Notes:

1. Congruence conditions for triangles are virtually identical to similarity conditions.
 - For conditions SSS and SAS, sides must be *congruent* rather than *proportional*.
 - Use **ASA** instead of **AA**: Triangles are congruent if they have 1 pair of congruent sides between 2 pairs of congruent corresponding angles.
2. Before referring to the minimal conditions of similarity or congruence, it is sometimes possible to find missing measures of triangles using the cosine and sine laws. However, this is generally not necessary.

Section B

Question 3

Find the coordinates of the vertex of a parabola that passes through points (1,8), (2,-2) and (7,8).

Answer: The vertex coordinates are **(4, -10)**.

Explanations

1. Calculate parameter h

The 1st and 3rd points have the same y-coordinate, which means that parameter h can be found by averaging their x-coordinates.

$$\begin{aligned}
 h &= \frac{x_1 + x_3}{2} \\
 &= \frac{1 + 7}{2} \\
 &= 4
 \end{aligned}$$

For now, we get the following rule in functional form.

$$f(x) = a(x - 4)^2 + k$$

2. Create a system of equations using 2 points

Be sure your system of equations does not include 2 points with the same y -coordinate. Let's use the 1st and 2nd points.

$$y_1 = a(x_1 - 4)^2 + k \quad y_2 = a(x_2 - 4)^2 + k$$

$$8 = a(1 - 4)^2 + k \quad -2 = a(2 - 4)^2 + k$$

$$8 = 9a + k \quad -2 = 4a + k$$

3. Solve the system of equations to find parameters a and k

We can use the reduction method to eliminate parameter k .

$$\begin{array}{r} 8 = 9a + k \\ -(-2 = 4a + k) \\ \hline 10 = 5a + 0 \\ 2 = a \end{array}$$

Now that we know the value of parameter a , we can calculate that of k .

$$8 = 9a + k$$

$$8 = 9(2) + k$$

$$8 = 18 + k$$

$$-10 = k$$

This gives us the following rule:

$$f(x) = 2(x - 4)^2 - 10$$

4. Enter the vertex coordinates

Answer: The vertex coordinates are $(4, -10)$.

Question 4

The area of a quadrilateral is represented by the following algebraic expression:

$$25x^2 - 35x + 12$$

Identify the type of quadrilateral and the binomials with integer coefficients corresponding to the lengths of its sides.

Answer: The sides of this **rectangle** are represented by the expressions **$5x - 4$** and **$5x - 3$** .

Explanations

To find the expressions corresponding to the sides of the quadrilateral, we need to factor the polynomial $25x^2 - 35x + 12$. Several factoring methods are possible. Let's use the *product-sum* technique.

Product-sum

$$25x^2 - 35x + 12$$

$$\begin{aligned} \text{Product} &= 25 \times 12 & \text{Sum} &= -35 \\ &= 300 \end{aligned}$$

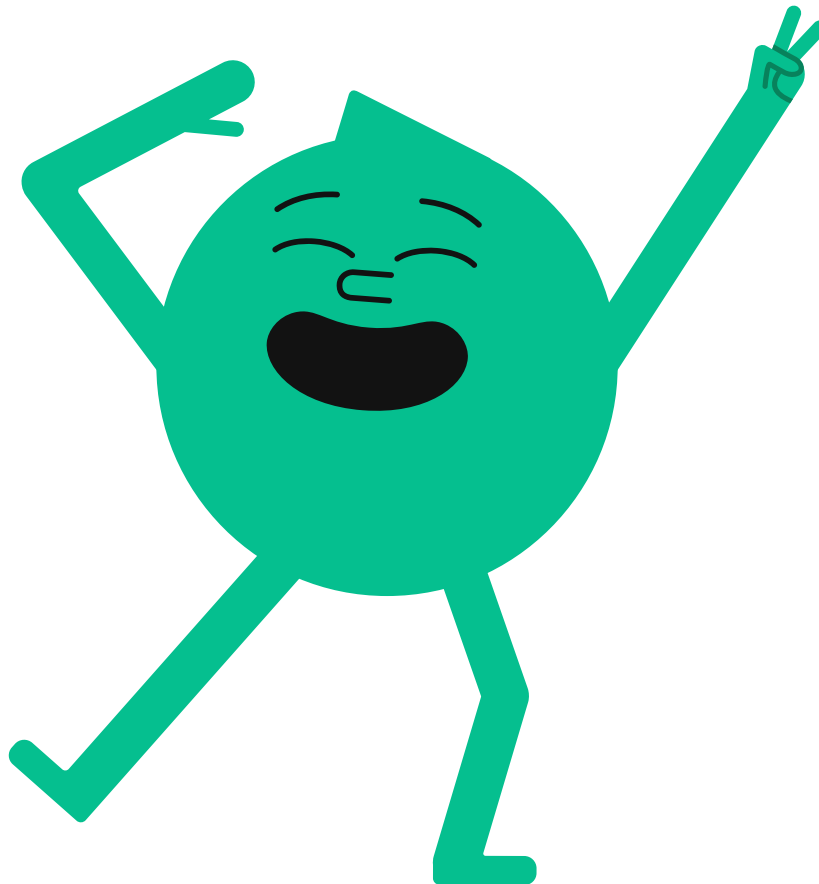
We'll use -20 and -15.

$$25x^2 - 20x - 15x + 12$$

$$5x(5x - 4) - 3(5x - 4)$$

$$(5x - 4)(5x - 3)$$

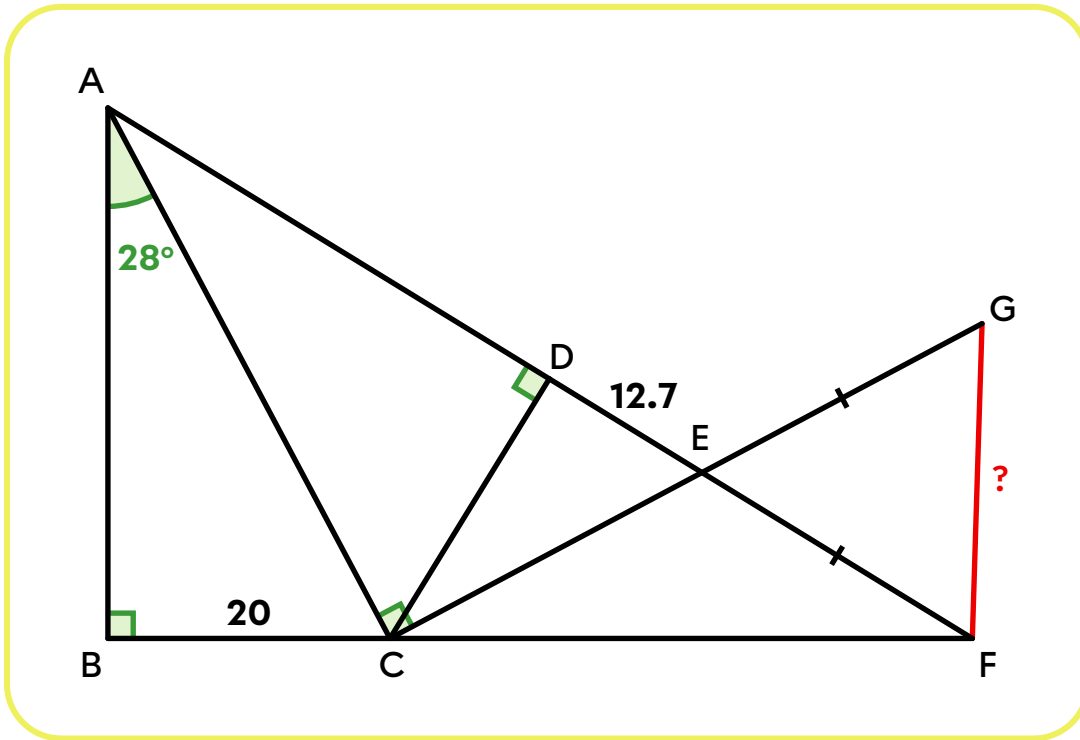
Because the 2 expressions we get differ for all x values, we can conclude that the quadrilateral is a **rectangle** whose sides are $5x - 4$ and $5x - 3$.



Section C

Question 5

What is the length of segment \overline{FG} ?



Several procedures are possible. Here's one.

1. Calculate the length of segment \overline{AC}

$$\sin(A) = \frac{m\overline{BC}}{m\overline{AC}}$$

$$\sin(28^\circ) = \frac{20}{m\overline{AC}}$$

$$m\overline{AC} = \frac{20}{\sin(28^\circ)}$$

$$m\overline{AC} \approx 42.6$$

2. Calculate the length of segment \overline{AD}

Each leg (\overline{AC}) of a right triangle ($\triangle ACE$) is the proportional mean between its projection onto the hypotenuse (\overline{AD}) and the entire hypotenuse (\overline{AE}).

Let's say $x = m\overline{AD}$.

$$\frac{m\overline{AD}}{m\overline{AC}} = \frac{m\overline{AC}}{m\overline{AE}}$$

$$m\overline{AC}^2 = m\overline{AD} \times m\overline{AE}$$

$$42.6^2 = x(x + 12.7)$$

$$1814.76 = x^2 + 12.7x$$

$$0 = x^2 + 12.7 - 1814.76$$

We then use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-12.7 \pm \sqrt{12.7^2 - 4(1)(-1814.76)}}{2(1)}$$

$$x_1 \approx 36.72 \quad x_2 \approx -49.42$$

x_2 can be ruled out because a segment length cannot be negative.
Thus, segment \overline{AD} measures approximately 36.72 units.

3. Calculate the length of segment \overline{CD}

In a right triangle ($\triangle ACE$), the height drawn from the right angle (\overline{CD}) is the proportional mean between the 2 segments it creates on the hypotenuse (\overline{AD} and \overline{DE}).

$$\frac{m\overline{AD}}{m\overline{CD}} = \frac{m\overline{CD}}{m\overline{DE}}$$

$$m\overline{CD}^2 = m\overline{AD} \times m\overline{DE}$$

$$m\overline{CD} = \sqrt{36.72 \times 12.7}$$

$$m\overline{CD} \approx 21.59$$

4. Calculate the length of segment \overline{CE}

In a right triangle ($\triangle ACE$), the product of the hypotenuse (\overline{AE}) and the corresponding height (\overline{CD}) is equal to the product of the legs (\overline{AC} and \overline{CE}).

$$m\overline{AE} \times m\overline{CD} = m\overline{AC} \times m\overline{CE}$$

$$(m\overline{AD} + m\overline{DE}) \times m\overline{CD} = m\overline{AC} \times m\overline{CE}$$

$$(36.72 + 12.7) \times 21.59 = 42.6 \times m\overline{CE}$$

$$m\overline{CE} = \frac{49.42 \times 21.59}{42.6}$$

$$m\overline{CE} \approx 25.05$$

5. Calculate the measures of angles DEC, ACB, ECF, CEF, CFE, and GEF

Triangle DEC is right-angled at D, so we can use a trigonometric ratio.

$$\begin{aligned}\tan(\angle DEC) &= \frac{m\overline{CD}}{m\overline{DE}} \\ &= \frac{21.59}{12.7} \\ m\angle DEC &= \tan^{-1}\left(\frac{21.59}{12.7}\right) \\ &\approx 59.5^\circ\end{aligned}$$

The sum of a triangle's interior angles equals 180° , so we can find the measure of angle ACB.

$$\begin{aligned}m\angle ACB &= 180^\circ - m\angle ABC - m\angle CAB \\ m\angle ACB &= 180^\circ - 90^\circ - 28^\circ \\ m\angle ACB &= 62^\circ\end{aligned}$$

Because angles ACB, ACE, and ECF form a straight angle (180°), we can find the measure of angle ECF.

$$\begin{aligned}m\angle ECF &= 180^\circ - m\angle ACB - m\angle ACE \\ m\angle ECF &= 180^\circ - 62^\circ - 90^\circ \\ m\angle ECF &= 28^\circ\end{aligned}$$

Angles DEC and CEF also form a straight angle.

$$\begin{aligned}m\angle CEF &= 180^\circ - m\angle DEC \\ &= 180^\circ - 59.5^\circ \\ &= 120.5^\circ\end{aligned}$$

The sum of a triangle's interior angles equals 180° , so we can find the measure of angle CFE.

$$\begin{aligned}m\angle CFE &= 180^\circ - m\angle CEF - m\angle ECF \\ &= 180^\circ - 120.5^\circ - 28^\circ \\ &= 31.5^\circ\end{aligned}$$

Angles DEC and GEF are opposite the vertex, so they are congruent.

$$m\angle GEF = m\angle DEC = 59.5^\circ$$

6. Calculate the lengths of segments \overline{EF} and \overline{EG}

We can use the sine law in triangle CEF.

$$\frac{m\overline{EF}}{\sin(\angle ECF)} = \frac{m\overline{CE}}{\sin(\angle CFE)}$$

$$\frac{m\overline{EF}}{\sin(28^\circ)} = \frac{25.05}{\sin(31.5^\circ)}$$

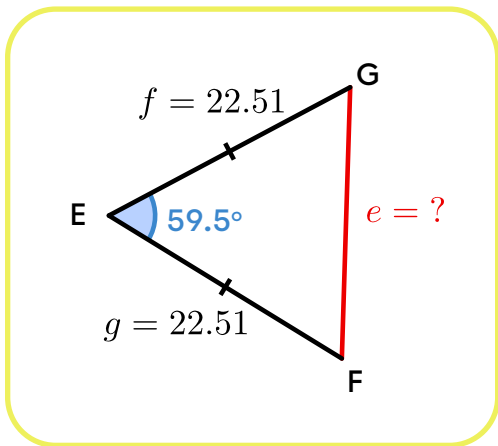
$$m\overline{EF} = \frac{25.05 \times \sin(28^\circ)}{\sin(31.5^\circ)}$$

$$\approx 22.51$$

The figure shows that segments \overline{EF} and \overline{EG} are congruent, so segment \overline{EG} also measures 22.51 units.

7. Calculate the length of segment \overline{FG}

We can use the law of cosines in triangle GEF.



$$e^2 = f^2 + g^2 - 2fg \cos(E)$$

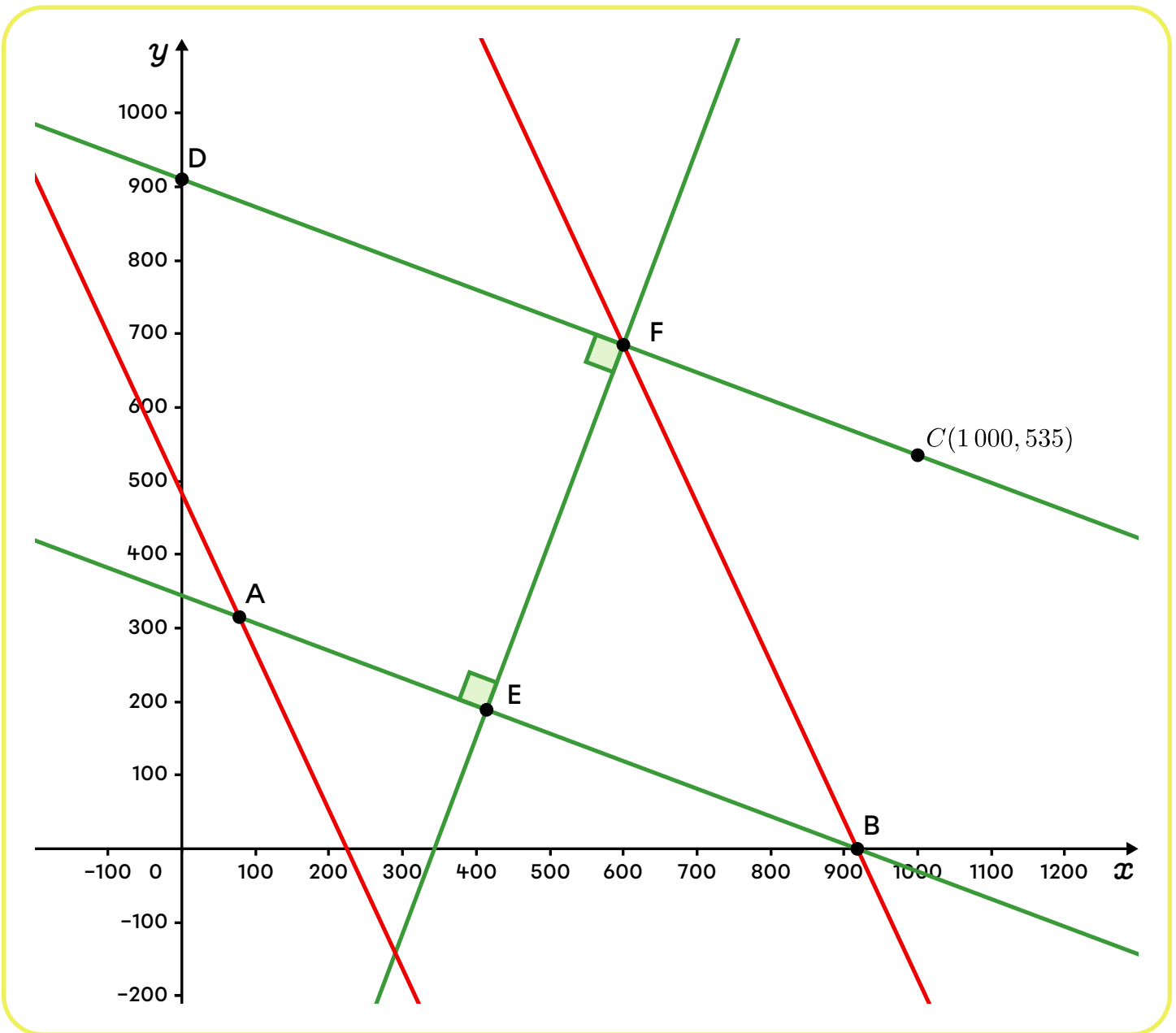
$$m\overline{FG}^2 = 22.51^2 + 22.51^2 - 2(22.51)(22.51) \cos(59.5^\circ)$$

$$m\overline{FG} = \sqrt{499.06}$$

$$\approx 22.34$$

Answer: Segment \overline{FG} measures approximately **22.34** units.

Question 6



There are two routes from point B to point D: route BFD and route BEFD. To determine the fastest route, calculate the length and duration of each trip, taking speed limits into account. To do this, first determine the coordinates of each point on the graph.

1. Find the coordinates of the points

a) Find the coordinates of point A

In the rule for line AB, replace x with 78 and isolate y .

$$3x + 8y = 2\,754$$

$$3(78) + 8y = 2\,754$$

$$234 + 8y = 2\,754$$

$$8y = 2\,520$$

$$y = 315$$

The coordinates of point A are (78, 315).

b) Find the coordinates of point B

Point B lies on the x -axis, so its y -coordinate is 0.

$$3x + 8y = 2\,754$$

$$3x + 8(0) = 2\,754$$

$$3x = 2\,754$$

$$x = 918$$

Therefore, the coordinates of point B are (918, 0).

c) Find the coordinates of point E

The y -coordinate of point A is 315. The y -coordinate of point E is equal to $\frac{3}{5}$ of the y -coordinate of point A, so replace y by $\frac{3}{5} \times 315 = 189$ in the rule for line AB.

$$3x + 8y = 2\,754$$

$$3x + 8(189) = 2\,754$$

$$3x + 1\,512 = 2\,754$$

$$3x = 1\,242$$

$$x = 414$$

Therefore, the coordinates of point E are (414, 189).

d) Determine the slope of line AB

Simply convert the rule for line AB from the general form to the functional form.

$$3x + 8y = 2754$$

$$8y = -3x + 2754$$

$$y = -\frac{3}{8}x + 344.25$$

Therefore, the slope of line AB is $-\frac{3}{8}$.

e) Calculate the rule for line EF

Lines EF and AB are perpendicular, so the slope of EF is the negative reciprocal of the slope of AB. In other words, the product of their slopes would be -1.

$$a_{AB} \times a_{EF} = -1$$

$$a_{EF} = \frac{-1}{a_{AB}}$$

$$a_{EF} = \frac{-1}{-\frac{3}{8}}$$

$$a_{EF} = \frac{8}{3}$$

Using the slope of EF and the coordinates of point E, we can find the rule for line EF.

$$y = ax + b$$

$$y = \frac{8}{3}x + b$$

$$189 = \frac{8}{3}(414) + b$$

$$189 = 1104 + b$$

$$-915 = b$$

The rule for line EF is $y = \frac{8}{3}x - 915$.

f) Determine the rule for line CD and the coordinates of point D

Because lines AB and CD are both perpendicular to line EF, they are parallel to each other. This also means that AB and CD have the same slope.

$$a_{AB} = a_{CD} = -\frac{3}{8}$$

Using the slope of line CD and the coordinates of point C, we can find the rule for line CD.

$$y = ax + b$$

$$y = -\frac{3}{8}x + b$$

$$535 = -\frac{3}{8}(1\,000) + b$$

$$535 = -375 + b$$

$$910 = b$$

The rule for line CD is $y = -\frac{3}{8}x + 910$.

→ Because point D lies on the y-axis, the y-intercept of line CD corresponds to the y-coordinate of point D. Thus, the coordinates of point D are (0, 910).

g) Find the coordinates of point F

Point F lies at the intersection of lines CD and EF. To find the coordinates of this intersection point, we can use the comparison method.

$$y_{CD} = y_{EF}$$

$$-\frac{3}{8}x + 910 = \frac{8}{3}x - 915$$

$$910 + 915 = \frac{8}{3}x + \frac{3}{8}x$$

$$1\,825 = \frac{73}{24}x$$

$$600 = x$$

Replace x with 600 in the rule for line CD to find the y-coordinate of point F.

$$y = -\frac{3}{8}x + 910$$

$$y = -\frac{3}{8}(600) + 910$$

$$y = 685$$

The coordinates of point F are (600, 685).

Note: It is also possible to replace x with 600 in the rule for line EF.

2. Calculate the lengths of segments BE, BF, EF, and DF

We can use the formula for finding the distance between 2 points.

$\text{dist}(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	
$\text{dist}(B, E) = \sqrt{(414 - 918)^2 + (189 - 0)^2}$ $m\overline{BE} \approx 538.27 \text{ m}$	$\text{dist}(E, F) = \sqrt{(600 - 414)^2 + (685 - 189)^2}$ $m\overline{EF} \approx 529.73 \text{ m}$
$\text{dist}(B, F) = \sqrt{(600 - 918)^2 + (685 - 0)^2}$ $m\overline{BF} \approx 755.21 \text{ m}$	$\text{dist}(D, F) = \sqrt{(600 - 0)^2 + (685 - 910)^2}$ $m\overline{DF} \approx 640.80 \text{ m}$

3. Calculate the duration of each trip

To calculate the duration of each trip, use the speed formula provided.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \iff \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Possible routes include $B \rightarrow F \rightarrow D$ and $B \rightarrow E \rightarrow F \rightarrow D$.

a) For route $B \rightarrow F \rightarrow D$, remember that segment BF is limited to 30 km/h, while segment FD is limited to 50 km/h. The distances calculated above were in metres (m), so convert them into kilometres (km).

$$\begin{aligned} \text{Time} &= \frac{\text{distance}(B, F)}{\text{speed}(B, F)} + \frac{\text{distance}(F, D)}{\text{speed}(F, D)} \\ &= \frac{0.75521 \text{ km}}{30 \text{ km/h}} + \frac{0.64080 \text{ km}}{50 \text{ km/h}} \\ &\approx 0.038 \text{ h} \\ &\approx 136.76 \text{ s} \end{aligned}$$

b) For route $B \rightarrow E \rightarrow F \rightarrow D$, speed is limited to 50 km/h.

$$\begin{aligned} \text{Time} &= \frac{\text{Total distance}}{\text{Speed}} \\ &= \frac{(0.53827 + 0.52973 + 0.64080) \text{ km}}{50 \text{ km/h}} \\ &= \frac{1.70880 \text{ km}}{50 \text{ km/h}} \\ &\approx 0.034 \text{ h} \\ &\approx 123.03 \text{ s} \end{aligned}$$

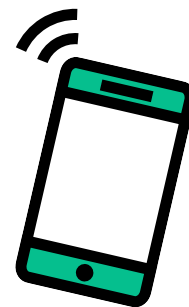
Conclusion: While route $B \rightarrow E \rightarrow F \rightarrow D$ is longer (in distance) than route $B \rightarrow F \rightarrow D$, it takes approximately 13.73 s less to complete.

Note: For more information on conjectures and proofs, consult the following pages.

Proofs



Conjectures



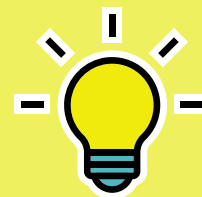
Question 7

Hint: The instructions could be reworded as follows: Using parameters a and b of function $f(x)$, how can we find the rule for function $g(x)$?

When formulating a conjecture, you should analyze at least 3 cases. Typically, we analyze the first case provided, then invent 2 more.

Tips

Work with a variety of examples. Choose whole numbers, decimal numbers, fractions, positive numbers, negative numbers, and so on. For example, if you use only positive integers, you risk not formulating a sufficiently precise conjecture.

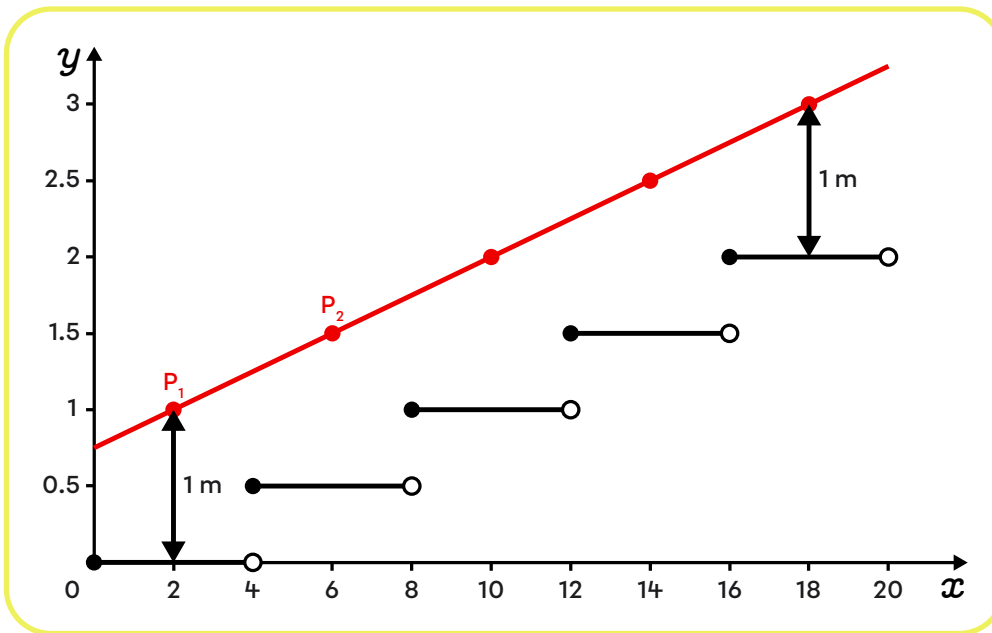


1st case: Parameters a and b are positive.

2nd case: Parameters a and b are negative.

3rd case: Parameter a is negative and parameter b is positive.

1st Case: Parameters a and b are positive



- Determine the value of parameters a and b of the staircase.
 - The height of each riser is 0.5 m. Therefore, $|a| = 0.5$.
 - The length of each step is 4 m. Therefore, $|b| = \frac{1}{4} = 0.25$.
 - The staircase ascends, and each step begins with a full point, so a and b are positive.

In summary, the rule is $f(x) = 0.5[0.25x]$.

- Determine the rule for the handrail ($g(x)$).

a) To determine the handrail rule, we need 2 points on the line.

→ Point P_1 is 1 m above the midpoint of the 1st step, which runs from 0 to 4, so the coordinates of P_1 are $(2, 1)$.

→ Point P_2 is 1 m above the midpoint of the 2nd step, which runs from 4 to 8 at a height of 0.5 m. Therefore, the coordinates of P_2 are $(6, 1.5)$.

b) Calculate the slope (m).

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1.5 - 1}{6 - 2} \\ &= \frac{1}{4} \end{aligned}$$

c) Determine the y-intercept (k).

$$g(x) = mx + k$$

$$g(x) = \frac{1}{8}x + k$$

$$1 = \frac{1}{8}(2) + k$$

$$\frac{3}{4} = k$$

Therefore, the rule is $g(x) = \frac{1}{8}x + \frac{3}{4}$.

2nd Case: Parameters a and b are negative

1. Choose the staircase rule ($f(x)$).

Let's opt for an **ascending** staircase with **negative** parameters **a** and **b** .

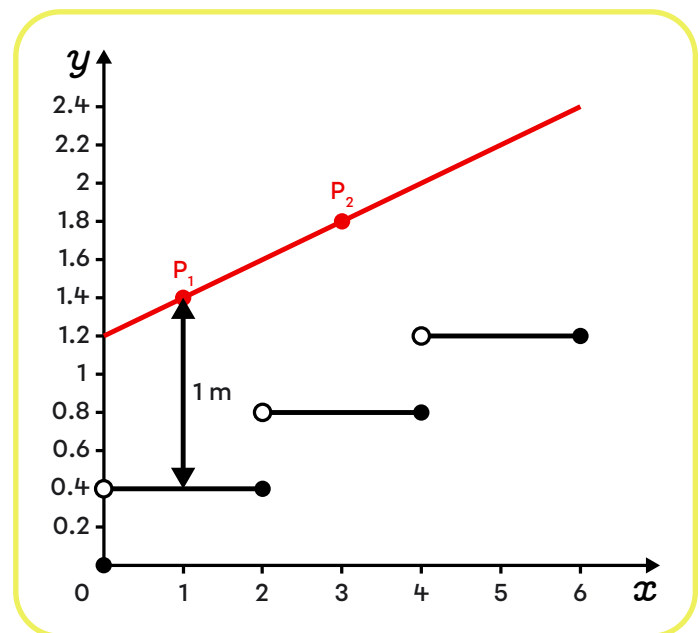
We'll use the following values:

→ If we select $a = -0.4$, each riser will be 0.4 m.

→ If $b = -0.5$, each step will have a length of $\frac{1}{0.5} = 2$ m.

→ Because we selected negative a and b parameters, the staircase will be ascending, and each step will start with an empty point.

The selected rule is therefore $f(x) = -0.4[-0.5x]$, and its graph is as follows:



2. Determine the rule for the handrail ($g(x)$).

a) To determine the handrail rule, we need 2 points on the line.

→ Point P_1 is 1 m above the midpoint of the 1st step, which runs from 0 to 2 at a height of 0.4 m. The coordinates of P_1 are therefore (1, 1.4).

→ Point P_2 is 1 m above the midpoint of the 2nd step, which runs from 2 to 4 at a height of 0.8 m. The coordinates of P_2 are therefore (3, 1.8).

b) Calculate the slope (m).

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1.8 - 1.4}{3 - 1} \\ &= 0.2 \end{aligned}$$

c) Determine the y-intercept (k).

$$g(x) = mx + k$$

$$g(x) = 0.2x + k$$

$$1.4 = 0.2(1) + k$$

$$1.2 = k$$

Therefore, the rule is $g(x) = 0.2x + 1.2$.

3rd Case: Parameter a is negative and parameter b is positive

1. Choose the staircase rule ($f(x)$).

Let's opt for a **descending** staircase with negative parameter a.

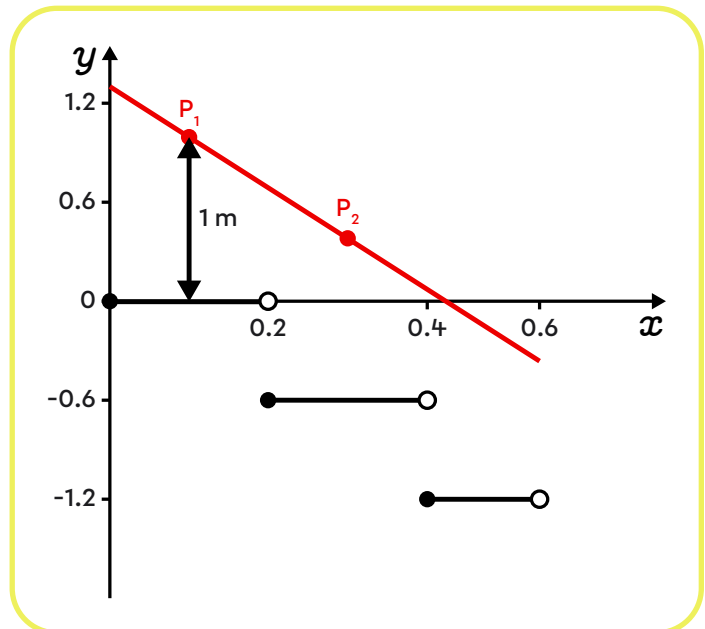
We'll use the following values:

→ If we select $a = -0.6$, each riser will be 0.6 m.

→ If $b = 5$, each step will have a length of $\frac{1}{5} = 0.2$ m.

→ Because a is negative and b is positive, the staircase will be descending, and each step will start with a full point.

The selected rule is therefore $f(x) = -0.6[5x]$, and its graph is as follows:



2. Determine the rule for the handrail ($g(x)$).

a) To determine the handrail rule, we need 2 points on the line.

→ Point P_1 is 1 m above the midpoint of the 1st step, which runs from 0 to 0.2 at a height of 0 m. Therefore, the coordinates of P_1 are $(0.1, 1)$.

→ Point P_2 is 1 m above the midpoint of the 2nd step, which runs from 0.2 to 0.4 at a height of -0.6 m. The coordinates of P_2 are therefore $(0.3, 0.4)$.

b) Calculate the slope (m).

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0.4 - 1}{0.3 - 0.1} \\ &= -3 \end{aligned}$$

c) Determine the y-intercept (k).

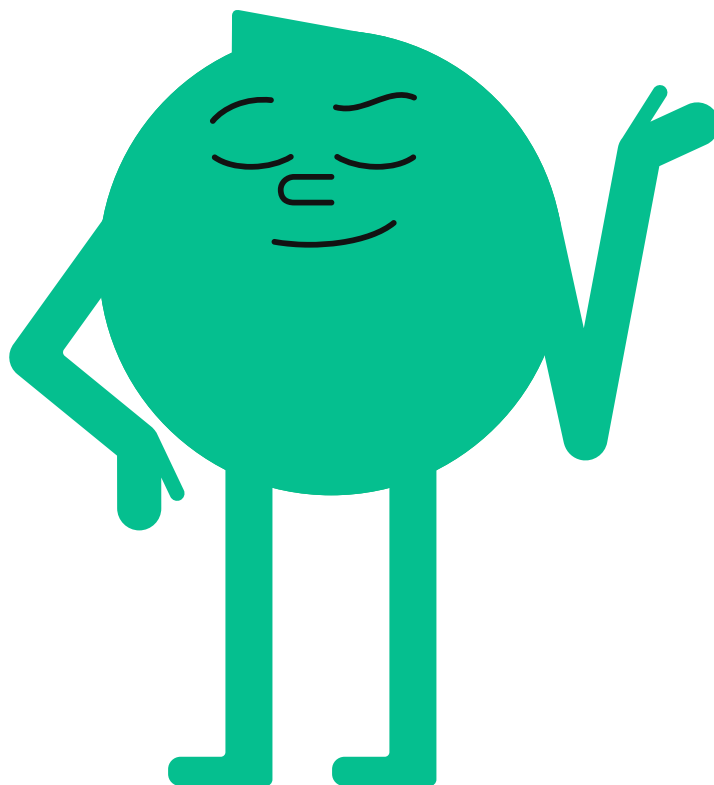
$$g(x) = mx + k$$

$$g(x) = -3x + k$$

$$1 = -3(0.1) + k$$

$$1.3 = k$$

Therefore, the rule is $g(x) = -3x + 1.3$.



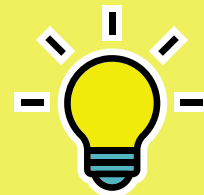
Looking for Relationships

Tips

It's often best to work with fractions to see relationships more clearly.

For example, it is easier to see that

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8} \text{ than to see that } 0.5 \times 0.25 = 0.125.$$



1st case	$a = 0.5 = \frac{1}{2}$ $b = 0.25 = \frac{1}{4}$	$m = \frac{1}{8}$ $k = \frac{3}{4}$
2nd case	$a = -0.4 = -\frac{2}{5}$ $b = -0.5 = -\frac{1}{2}$	$m = 0.2 = \frac{1}{5}$ $k = 1.2 = \frac{6}{5}$
3rd case	$a = -0.6 = -\frac{3}{5}$ $b = 5$	$m = -3$ $k = 1.3 = \frac{13}{10}$

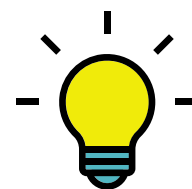
When we're looking for relationships, we often need to ask ourselves questions like the following:

• For m

- How can we get $\frac{1}{8}$ using $\frac{1}{2}$ and $\frac{1}{4}$?
- How can we get $\frac{1}{5}$ using $-\frac{2}{5}$ and $-\frac{1}{2}$?
- How can we get -3 using $-\frac{3}{5}$ and 5 ?

• For k

- How can we get $\frac{3}{4}$ using $\frac{1}{2}$ and $\frac{1}{4}$ (without forgetting the 1 m)?
- How can we get 1.2 using -0.4 and -0.5 (without forgetting the 1 m)?
- How can we get 1.3 using -0.6 and 5 (without forgetting the 1 m)?



Tips

- Try basic operations first (addition, subtraction, multiplication, and division).

Examples

$$\frac{1}{2} + \frac{1}{4} \stackrel{?}{=} \frac{1}{8} \quad \text{⊗}$$

$$\frac{1}{2} - \frac{1}{4} \stackrel{?}{=} \frac{1}{8} \quad \text{⊗}$$

$$\frac{1}{2} \times \frac{1}{4} \stackrel{?}{=} \frac{1}{8} \quad \text{⊙}$$

$$\frac{1}{2} \div \frac{1}{4} \stackrel{?}{=} \frac{1}{8} \quad \text{⊗}$$

Verify that **the relationship you've found** also works in the other 2 cases.

- Try to use only one parameter at a time.

Examples

	With parameter b	With parameter a
1st case	$1 - b = 1 - \frac{1}{4} = \frac{3}{4} = k$ <p style="text-align: center;">⊙</p>	$1 - \frac{1}{2}a = 1 - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} = k$ <p style="text-align: center;">⊙</p>
2nd case	$1 - b = 1 - (-0.5) = 1.5 \neq k$ <p style="text-align: center;">⊗</p>	$1 - \frac{1}{2}a = 1 - \frac{1}{2} \times -0.4 = 1.2 = k$ <p style="text-align: center;">⊙</p>

- Analyze the graph and use your mathematical knowledge.

Examples

- The y-intercept of a linear function (**k**) is related to the *vertical scale*, as is parameter **a** of a greatest integer function.
- To determine parameter **k**, the *height* of the handrail from the midpoint of the step (**1 m**) must also be taken into account.

Relevant relationships:

• **For m**

In all cases, multiplying parameters a and b gives parameter m .

- 1st case: $ab = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} = m$
- 2nd case: $ab = -0.4 \times -0.5 = 0.2 = m$
- 3rd case: $ab = -\frac{3}{5} \times 5 = -3 = m$

We verified that it works equally well in all cases, i.e., with positive, negative, and opposing sign a and b parameters.

• **For k**

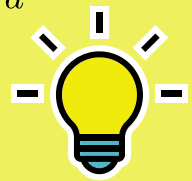
In all 3 cases, subtracting half the value of parameter a to the 1 m height gives the following:

- 1st case: $1 - \frac{1}{2}a = 1 - \frac{1}{2}\left(\frac{1}{2}\right) = 1 - \frac{1}{4} = \frac{3}{4} = k$
- 2nd case: $1 - \frac{1}{2}a = 1 - \frac{1}{2}(-0.4) = 1 - (-0.2) = 1.2 = k$
- 3rd case: $1 - \frac{1}{2}a = 1 - \frac{1}{2}(-0.6) = 1 - (-0.3) = 1.3 = k$

We're now ready to formulate a complete answer.

Tips

- You must write your answer, i.e., your conjecture, using complete sentences. In other words, while " $m = a \times b$ " and " $k = 1 - \frac{1}{2}a$ " can be included in your conjecture, this is not the expected answer.
- It's better to respond with a relationship that doesn't work in every case than none at all.



Answer

- The slope of function $g(x)$ is equal to the product of parameters a and b of function $f(x)$, regardless of the sign of a and b .

In algebraic form: $m = a \times b$

- The y -intercept k of function $g(x)$ is obtained by subtracting half the value of parameter a to the 1 m height.

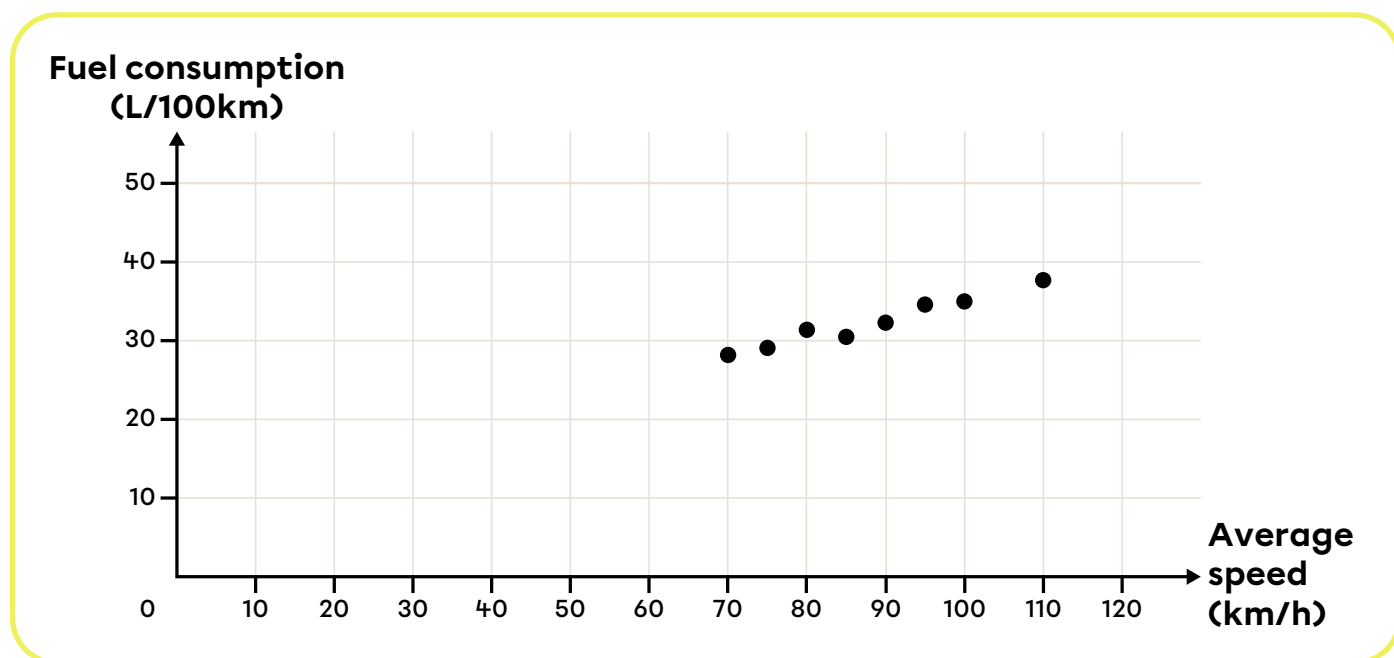
In algebraic form: $k = 1 - \frac{1}{2}a$

Question 8

The main steps in solving the problem are as follows: We need to determine the regression line equation for the MetaCargo fleet data, then find the rule for fuel consumption as a function of average speed for the NovaFret fleet. Next, we need to use various speeds to analyze which truck fleet consumes less fuel.

A. Let's start with the **MetaCargo** fleet.

We create a scatter plot to see if there's a correlation between the variables.



There is a positive correlation between the variables, meaning the higher a truck's average speed, the higher its fuel consumption.

So, we're going to find the regression line rule. We can use either the **Mayer method** or median-median method. Let's use the Mayer method.

- Order coordinates according to the independent variable.
This has already been done.

x : Average speed (km/h)	70	75	80	85	90	95	100	110
y : Fuel consumption (L/100 km)	28.2	29.1	31.4	30.5	32.3	34.6	35.0	37.7

2. Separate the distribution into 2 equal groups if possible.

	1st group				2nd group			
x : Average speed (km/h)	70	75	80	85	90	95	100	110
y : Fuel consumption (L/100 km)	28.2	29.1	31.4	30.5	32.3	34.6	35.0	37.7

3. Calculate the **average point** for each group (P_1 and P_2).

	1st group	2nd group
Average x-coordinate	$\bar{x}_1 = \frac{70 + 75 + 80 + 85}{4} = 77.5$	$\bar{x}_2 = \frac{90 + 95 + 100 + 110}{4} = 98.75$
Average y-coordinate	$\bar{y}_1 = \frac{28.2 + 29.1 + 31.4 + 30.5}{4} = 29.8$	$\bar{y}_2 = \frac{32.3 + 34.6 + 35.0 + 37.7}{4} = 34.9$
Average point	$P_1 (77.5, 29.8)$	$P_2 (98.75, 34.9)$

4. Find the rule of the regression line ($y = ax + b$) passing through points P_1 and P_2 .

a) Calculate the slope.

$$\begin{aligned}
 a &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{34.9 - 29.8}{98.75 - 77.5} \\
 &= \frac{5.1}{21.25} \\
 &= 0.24
 \end{aligned}$$

Therefore, $y = 0.24x + b$.

b) Calculate the value of b using point $P_1 (77.5, 29.8)$.

$$y = 0.24x + b$$

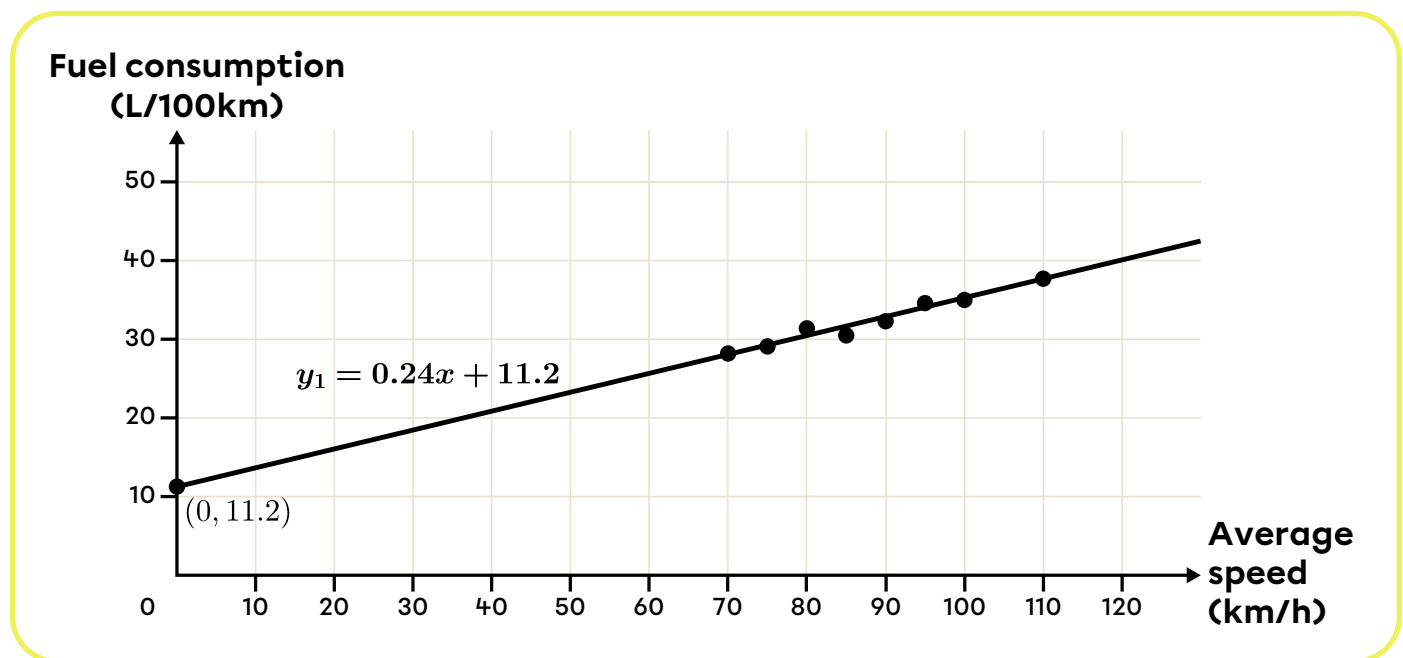
$$29.8 = 0.24(77.5) + b$$

$$11.2 = b$$

Thus, the regression line rule for MetaCargo is $y_1 = 0.24x + 11.2$.

Note: If you used the median-median method, you should get $y_1 = 0.236x + 11.18\bar{3}$.

We can now plot the regression line as it passes through a y -intercept of 11.2 and the scatter plot.



B. Now let's find the rule for the function that calculates fuel consumption for **NovaFret** trucks.

- Use the model $y = ax^2 + k$.
- Use the point $(0, 14.35)$.
- Use the point $(75, 25.6)$ as well.

In the rule, replace x by 0 and y by 14.35.

$$y = ax^2 + k$$

$$14.35 = a(0)^2 + k$$

$$14.35 = k$$

Because $h = 0$, point $(0, 14.35)$ is the vertex of the parabola.

Using parameter k , we can now calculate parameter a by replacing x by 75 and y by 25.6.

$$y = ax^2 + 14.35$$

$$25.6 = a(75)^2 + 14.35$$

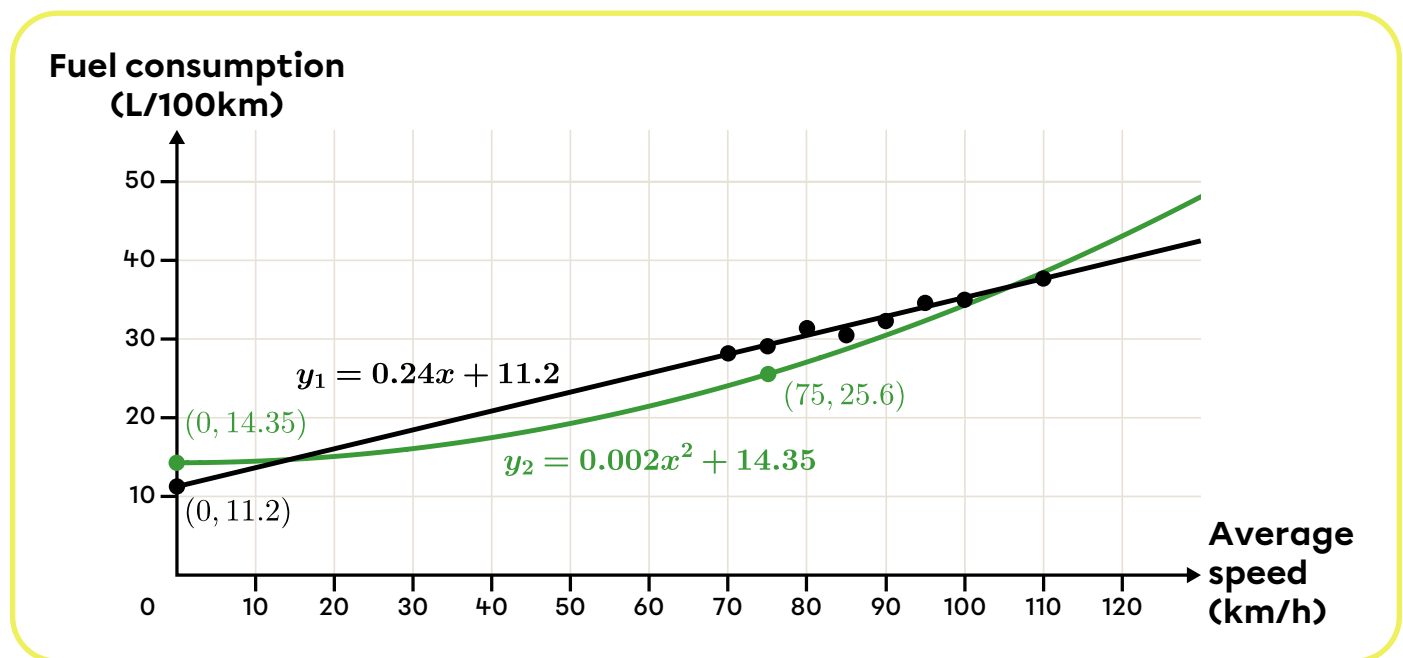
$$11.25 = 5625a$$

$$0.002 = a$$

Thus, the rule for calculating fuel consumption for trucks in the NovaFret fleet (y_2) as a function of average speed (x) in km/h is $y_2 = 0.002x^2 + 14.35$.

We can now plot the curve of the y_2 function in the same Cartesian plane. We know the following:

- The vertex is located at $(0, 14.35)$.
- The parabola opens at the top because parameter a is positive.
- The parabola passes through point $(75, 25.6)$.



- C. To advise the company president properly, we need to find the average speed (s) at which both truck fleets consume the same amount of fuel. Therefore, we should find the points where the line and curve intersect.

We can use the method of our choice: comparison, substitution, or reduction. The y variable is already isolated in the 2 equations, so let's use the comparison method.

$$y_1 = y_2$$

$$0.24x + 11.2 = 0.002x^2 + 14.35$$

$$\frac{0}{0.002} = \frac{0.002x^2 - 0.24x + 3.15}{0.002}$$

$$0 = x^2 - 120x + 1575$$

Once the equation equals 0, we can find the values of x by using the quadratic formula or by factoring. We'll do that here by **completing the square**.

$$0 = x^2 - 120x + \left(\frac{120}{2}\right)^2 - \left(\frac{120}{2}\right)^2 + 1\,575$$

$$0 = (x^2 - 120x + 3\,600) - 3\,600 + 1\,575$$

$$0 = (x^2 - 2(60)x + 60^2) - 2\,025$$

$$0 = (x - 60)^2 - 45^2$$

$$0 = (x - 60 + 45)(x - 60 - 45)$$

$$0 = (x - 15)(x - 105)$$

$$\begin{array}{cc} \downarrow & \downarrow \\ x_1 = 15 & x_2 = 105 \end{array}$$

Note: If you have found the rule for y_1 using the median-median method, you should get $x_1 \approx 115.44$ et $x_2 \approx 102.56$.

So, according to the data available, the trucks in these two fleets consume the same amount of fuel when travelling at average speeds of 15 km/h and 105 km/h. To see what happens when speeds drop below 15 km/h, stay between 15 km/h and 105 km/h, or exceed 105 km/h, we can look at the graph or analyse specific values within each interval.

	MetaCargo	NovaFret
[0, 15[km/h → Assume $x = 10$	$y_1 = 0.24x + 11.2$ $= 0.24(10) + 11.2$ $= 13.6 \text{ L/100 km}$	$y_2 = 0.002x^2 + 14.35$ $= 0.002(10)^2 + 14.35$ $= 14.55 \text{ L/100 km}$
15 km/h	14.8 L/100 km	
]15, 105[km/h → Assume $x = 100$	$y_1 = 0.24x + 11.2$ $= 0.24(100) + 11.2$ $= 35.2 \text{ L/100 km}$	$y_2 = 0.002x^2 + 14.35$ $= 0.002(100)^2 + 14.35$ $= 34.35 \text{ L/100 km}$
105 km/h	36.4 L/100 km	
Over de 105 km/h → Assume $x = 120$	$y_1 = 0.24x + 11.2$ $= 0.24(120) + 11.2$ $= 40 \text{ L/100 km}$	$y_2 = 0.002x^2 + 14.35$ $= 0.002(120)^2 + 14.35$ $= 43.15 \text{ L/100 km}$

- When speeds stay between 15 km/h and 105 km/h, function y_2 shows that the NovaFret fleet consumes the least amount of fuel.
- When average speeds stay below 15 km/h or exceed 105 km/h, MetaCargo trucks offer the best fuel economy.

Answer:

Here are 2 ways of formulating the answer:

- If the average speed on a given route stays **between 15 km/h and 105 km/h**, he should use trucks from the **MétaCargo** fleet. Otherwise, he should go with the **NovaFret** fleet.
- If the average speed on a given route stays **below 15 km/h or above 105 km/h**, he should use trucks from the **NovaFret** fleet. Otherwise, he should go with the **MétaCargo** fleet.

