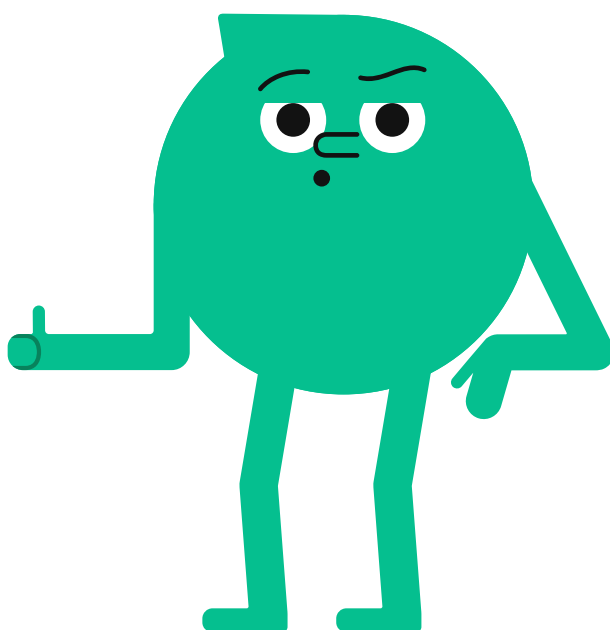


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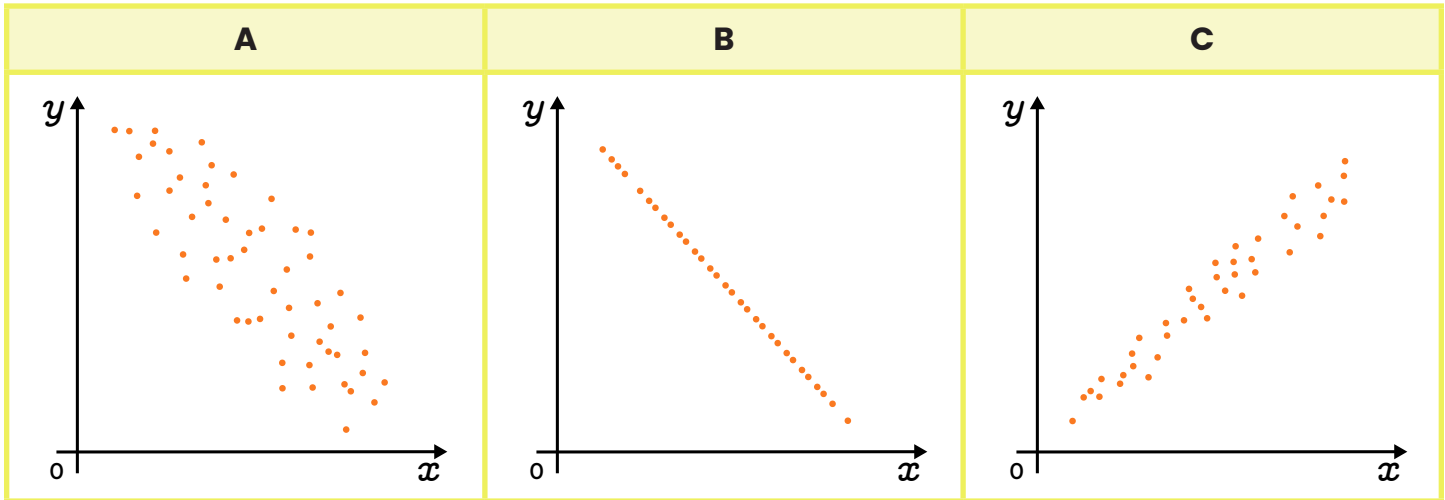


Answer Key

Section A

Question 1

Place the following scatter plots in **descending** order of linear correlation **coefficient**.



- a) $B > C > A$
 b) $A < C < B$
 c) $C > A > B$
 d) $B < A < C$

Answer: c) $C > A > B$

Explanations

Scatter plot **C** is the only graph displaying an **upward** trend. It is therefore the only one with a **positive** correlation coefficient. Because the remaining 2 are negative, C must necessarily have the highest correlation coefficient.

Tip: Because they should be arranged in *descending order*, the correct answer must begin with **C**. Therefore, **by process of elimination**, choices a), b), and d) can be ruled out.

Let's go into more detail. We'll start by estimating the correlation coefficients of the 3 scatter plots.

- Graph **A** shows a **weak negative** correlation. Therefore, $r_A \approx -0.6$.
- Graph **B** shows a **perfect negative** correlation. Therefore, $r_B = -1$.
- Graph **C** shows a **moderate positive** correlation. Therefore, $r_C \approx +0.8$.

Thus, in *descending order*, we get $+0.8 > -0.6 > -1$, i.e., answer **c) $C > A > B$** . Answer **d) $B < A < C$** is incorrect because it appears in *ascending* rather than *descending order*.

If we had been asked to list the scatter plots in order of correlation **strength**, we would have ignored the + and - signs.

- The scatter plot with the weakest correlation is graph A.
- Scatter plot C has a stronger correlation than scatter plot A.
- Scatter plot B has an even stronger correlation because it is perfect.

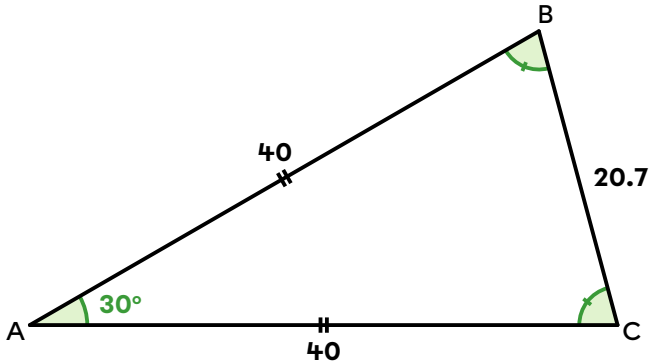
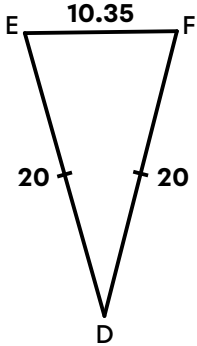
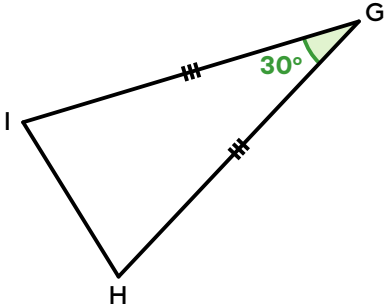
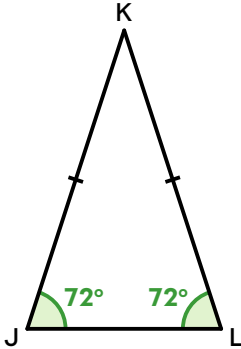
Thus, from weakest to strongest correlation (regardless of signs), the order is **b) $A < C < B$** . From strongest to weakest correlation, the order is reversed, i.e., option **a) $B > C > A$** .

Keep in mind: During the exam, be sure to pay attention to the order of the information (ascending or descending) and what must be put in order, e.g., the correlation **coefficients** (take the plus and minus signs into account) or correlation **strength** (ignore the signs).

Question 2

Which of these doesn't belong?

All but one of the following triangles are similar. Which one?

A	B
	
C	D
	

Answer: d) triangle JKL

- Triangle ABC is similar to triangle DEF by **SSS**: Triangles are similar if their corresponding sides are proportional.

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{BC}}{m\overline{EF}}$$

$$\frac{40}{20} = \frac{40}{20} = \frac{20.7}{10.35}$$

$$2 = 2 = 2$$

- Triangle ABC is similar to triangle GHI by **SAS**: Triangles are similar if they have 1 pair of congruent angles between 2 pairs of proportional sides.

$$\circ m\angle BAC = m\angle HGI = 30^\circ$$

$$\circ \frac{m\overline{AB}}{m\overline{GH}} = \frac{m\overline{AC}}{m\overline{GI}}$$

$$\frac{40}{m\overline{GH}} = \frac{40}{m\overline{GI}}$$

This equality is true because segments GH and GI are congruent.

Note: In these 2 triangles, the congruent angles are indeed located between the proportional sides.

Since triangle ABC is similar to triangle DEF **and** triangle GHI, we can assert, via the transitive property, that triangles DEF and GHI are similar to each other. In other words, we've already identified the 3 similar triangles. By process of elimination, the remaining triangle, KJL, doesn't belong. Here's why.

- Triangles ABC and KJL appear similar by AA (triangles are similar if they have 2 pairs of congruent angles), but they are not. Angles B and C are congruent to each other, as are angles J and L, but this doesn't mean that angle C is congruent to angle J. Let's see why.

In triangle ABC, angle A measures 30° . Since the sum of a triangle's interior angles equals 180° , we can calculate the measures of angles B and C.

$$m\angle B = m\angle C = \frac{180^\circ - 30^\circ}{2} = 75^\circ$$

In triangle KJL, angles J and L measure 72° , not 75° .

Notes:

1. Congruence conditions for triangles are virtually identical to similarity conditions.
 - For conditions SSS and SAS, sides must be *congruent* rather than *proportional*.
 - Use **ASA** instead of **AA**: Triangles are congruent if they have 1 pair of congruent sides between 2 pairs of congruent corresponding angles.
2. Before referring to the minimal conditions of similarity or congruence, it is sometimes possible to find missing measures of triangles using the cosine and sine laws. However, this is generally not necessary.

Section B

Question 3

Consider a parabola passing through the point (8, 48). What is the x-coordinate, in fractional notation, of the point or points on the parabola whose y-coordinate is $\frac{4}{27}$?

Answer: The point or points with the y-coordinate $\frac{4}{27}$ have the x-coordinate - 4 / 9 and 4 / 9.

Explanations

1. Find the rule of the parabola.

First, we calculate parameter a using point (8, 48).

$$\begin{aligned}
 f(x) &= ax^2 \\
 48 &= a(8)^2 \\
 48 &= 64a \\
 \frac{48}{64} &= a \\
 \frac{3}{4} &= a
 \end{aligned}$$

This gives us the following rule:

$$f(x) = \frac{3}{4}x^2$$

2. Calculate the x-coordinates when the y-coordinate is $\frac{4}{27}$.

We replace $f(x)$ by $\frac{4}{27}$, then isolate x .

$$f(x) = \frac{3}{4}x^2$$

$$\frac{4}{27} = \frac{3}{4}x^2$$

$$\frac{4}{27} \times \frac{4}{3} = x^2$$

$$\frac{16}{81} = x^2$$

$$\pm \sqrt{\frac{16}{81}} = x$$

$$\pm \frac{4}{9} = x$$

This gives us the points $(-\frac{4}{9}, \frac{4}{27})$ and $(\frac{4}{9}, \frac{4}{27})$

Question 4

Last year, Marie set up a new business for her Latin dance studio. Her business is doing well, and she wants to grow her customer base. She decides to conduct a statistical study of her current customers to better target potential customers with her advertising campaign. For this purpose, she compiles the ages of her dancers in the following stem-and-leaf plot.

Women		Men
9-5	2	
8-7-6-6-4-1	3	5-9
8-5-4-2	4	3-3-7-8
3-1	5	1-2-2-4

Help Marie calculate the average age of the women, the average age of the men, and the mean deviation for all of the dancers. Round your answers to the nearest tenth.

Answer:

- The mean age of the women is 39.2 years.
- The mean age of the men is 46.4 years.
- The mean deviation for the ages of the men and the women is 6.9 years.

Explanations

Remember: In a stem-and-leaf plot, each digit in the middle column (stem) corresponds to the first digit of a data value, while the digits in the other 2 columns correspond to the rest of the data value.

1. Calculate the mean age of the women

Let's start by making a list of all of the women's ages found in the stem-and-leaf plot.

25, 29, 31, 34, 36, 36, 37, 38, 42, 44, 45, 48, 51, 53

Now let's calculate the mean.

$$\begin{aligned}\bar{x}_{\text{women}} &= \frac{\sum x_i}{n_{\text{women}}} \\ &= \frac{25 + 29 + 31 + \dots + 48 + 51 + 53}{14} \\ &\approx 39.2\end{aligned}$$

The mean age of the women is approximately 39.2.

2. Calculate the mean age of the men

Let's start by making a list of all of the men's ages found in the stem-and-leaf plot.

35, 39, 43, 43, 47, 48, 51, 52, 52, 54

Now let's calculate the mean.

$$\begin{aligned}\bar{x}_{\text{men}} &= \frac{\sum x_i}{n_{\text{men}}} \\ &= \frac{35 + 39 + 43 + \dots + 52 + 52 + 54}{10} \\ &= 46.4\end{aligned}$$

The mean age of the men is approximately 46.4.

3. Calculate the mean deviation of all the dancers

The formula for calculating the mean deviation is as follows:

$$MD = \frac{\sum |x_i - \bar{x}|}{n}$$

Let's break down this calculation into 3 steps.

a) Calculate the overall mean $\bar{x}$

We can use the means calculated in steps 1 and 2 to calculate the overall mean. To do this, we can simply calculate a weighted mean of these 2 values, taking into account the number of men and women who take classes at the studio.

$$\begin{aligned}\bar{x} &= \frac{n_{\text{women}} \bar{x}_{\text{women}} + n_{\text{men}} \bar{x}_{\text{men}}}{n_{\text{women}} + n_{\text{men}}} \\ &= \frac{14 \times 39.2 + 10 \times 46.4}{14 + 10} \\ &= 42.2\end{aligned}$$

b) Calculate the deviations from the mean

Now that we know the overall mean $\bar{x}$, we can calculate the deviation from the mean for each data value in the distribution.

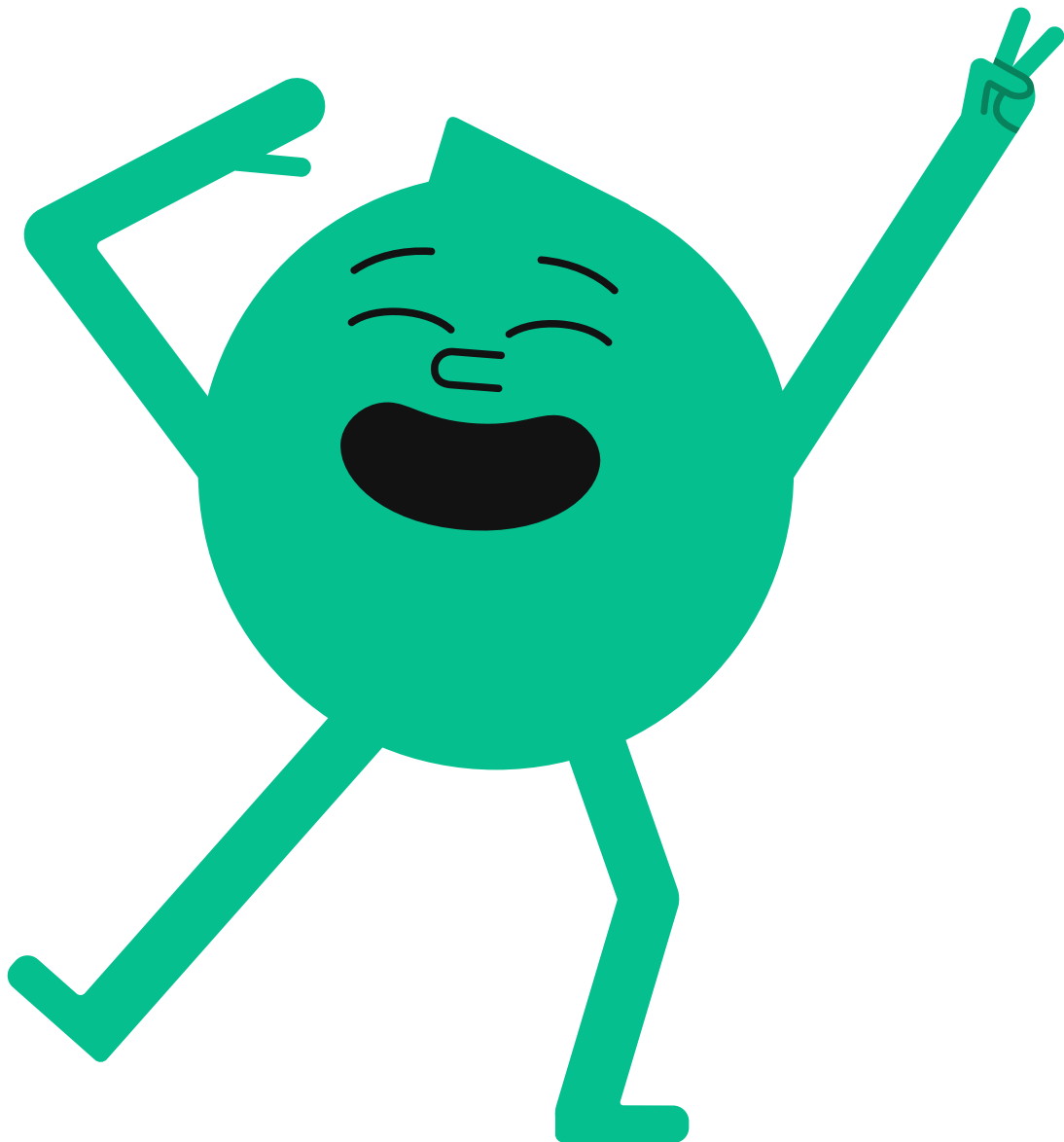
Women		Men	
Age x_i	Deviation from the mean $ x_i - \bar{x} $	Age x_i	Deviation from the mean $ x_i - \bar{x} $
25	$ 25 - 42.2 = 17.2$	35	$ 35 - 42.2 = 7.2$
29	$ 29 - 42.2 = 13.2$	39	$ 39 - 42.2 = 3.2$
31	$ 31 - 42.2 = 11.2$	43	$ 43 - 42.2 = 0.8$
34	$ 34 - 42.2 = 8.2$	43	$ 43 - 42.2 = 0.8$
36	$ 36 - 42.2 = 6.2$	47	$ 47 - 42.2 = 4.8$
36	$ 36 - 42.2 = 6.2$	48	$ 48 - 42.2 = 5.8$
37	$ 37 - 42.2 = 5.2$	51	$ 51 - 42.2 = 8.8$
38	$ 38 - 42.2 = 4.2$	52	$ 52 - 42.2 = 9.8$
42	$ 42 - 42.2 = 0.2$	52	$ 52 - 42.2 = 9.8$
44	$ 44 - 42.2 = 1.8$	54	$ 54 - 42.2 = 11.8$
45	$ 45 - 42.2 = 2.8$		
48	$ 48 - 42.2 = 5.8$		
51	$ 51 - 42.2 = 8.8$		
53	$ 53 - 42.2 = 10.8$		

c) Calculate the mean deviation

For the final calculation, add up all of the deviations from the mean, then divide by the total number of customers.

$$\begin{aligned}
 MD &= \frac{\sum |x_i - \bar{x}|}{n} \\
 &= \frac{17.2 + 13.2 + 11.2 + \dots + 9.8 + 9.8 + 11.8}{14 + 10} \\
 &\approx 6.9
 \end{aligned}$$

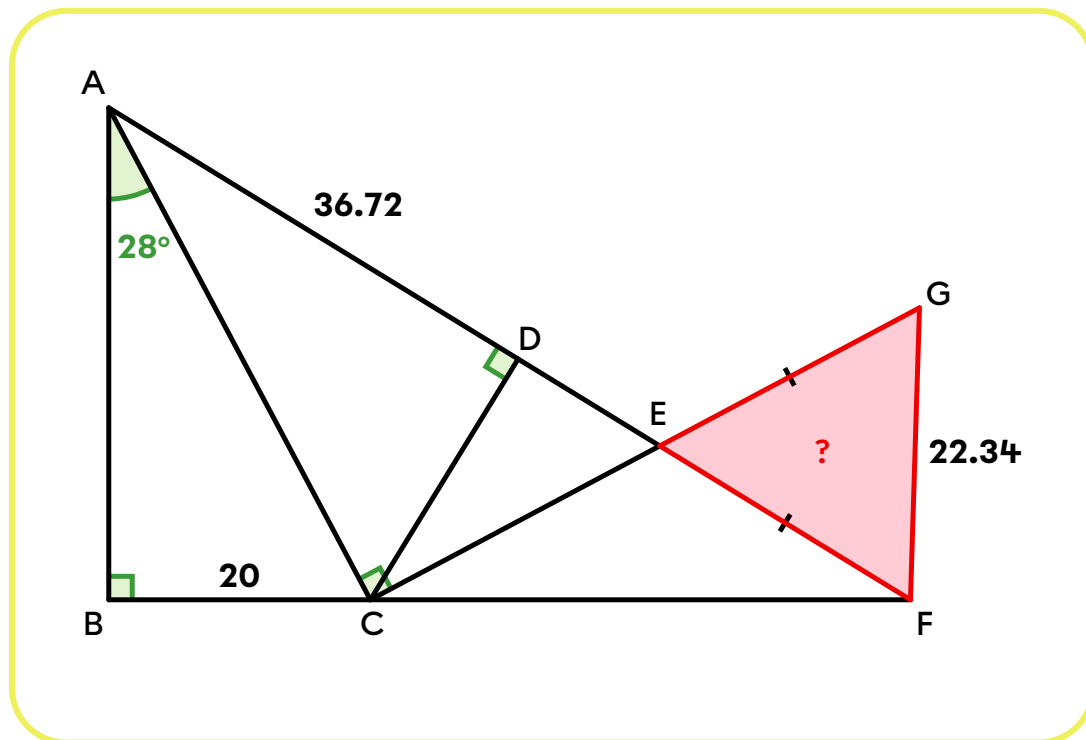
The mean deviation for the ages of the dancers at Marie's studio is approximately 6.9 years.



Section C

Question 5

What is the area of triangle GEF?



Several procedures are possible. Here's one.

1. Calculate the length of segment $\overline{AC}$

$$\sin(A) = \frac{m\overline{BC}}{m\overline{AC}}$$

$$\sin(28^\circ) = \frac{20}{m\overline{AC}}$$

$$m\overline{AC} = \frac{20}{\sin(28^\circ)}$$

$$m\overline{AC} \approx 42.6$$

2. Calculate the length of segment $\overline{AE}$

Each leg ($\overline{AC}$) of a right triangle ($\triangle ACE$) is the proportional mean between its projection onto the hypotenuse ($\overline{AD}$) and the entire hypotenuse ($\overline{AE}$).

$$\frac{m\overline{AD}}{m\overline{AC}} = \frac{m\overline{AC}}{m\overline{AE}}$$

$$m\overline{AC}^2 = m\overline{AD} \times m\overline{AE}$$

$$42.6^2 = 36.72 \times m\overline{AE}$$

$$49.42 \approx m\overline{AE}$$

3. Calculate the length of segment $\overline{DE}$

Points A, D, and E are aligned. Therefore, $m\overline{AE} = m\overline{AD} + m\overline{DE}$.

$$m\overline{DE} = m\overline{AE} - m\overline{AD}$$

$$= 49.42 - 36.72$$

$$= 12.7$$

4. Calculate the length of segment $\overline{CD}$

In a right triangle ($\triangle ACE$), the height drawn from the right angle ($\overline{CD}$) is the proportional mean between the 2 segments it creates on the hypotenuse ($\overline{AD}$ and $\overline{DE}$).

$$\frac{m\overline{AD}}{m\overline{CD}} = \frac{m\overline{CD}}{m\overline{DE}}$$

$$m\overline{CD}^2 = m\overline{AD} \times m\overline{DE}$$

$$m\overline{CD} = \sqrt{36.72 \times 12.7}$$

$$m\overline{CD} \approx 21.59$$

5. Calculate the length of segment $\overline{CE}$

In a right triangle ($\triangle ACE$), the product of the hypotenuse ($\overline{AE}$) and the corresponding height ($\overline{CD}$) is equal to the product of the legs ($\overline{AC}$ and $\overline{CE}$).

$$m\overline{AE} \times m\overline{CD} = m\overline{AC} \times m\overline{CE}$$

$$49.42 \times 21.59 = 42.6 \times m\overline{CE}$$

$$m\overline{CE} = \frac{49.42 \times 21.59}{42.6}$$

$$m\overline{CE} \approx 25.05$$

6. Calculate the measures of angles DEC, ACB, ECF, CEF and CFE

Triangle DEC is right-angled at D, so we can use a trigonometric ratio.

$$\begin{aligned}\tan(\angle DEC) &= \frac{m\overline{CD}}{m\overline{DE}} \\ &= \frac{21.59}{12.7} \\ m\angle DEC &= \tan^{-1}\left(\frac{21.59}{12.7}\right) \\ &\approx 59.5^\circ\end{aligned}$$

The sum of a triangle's interior angles equals 180° , so we can find the measure of angle ACB.

$$\begin{aligned}m\angle ACB &= 180^\circ - m\angle ABC - m\angle CAB \\ m\angle ACB &= 180^\circ - 90^\circ - 28^\circ \\ m\angle ACB &= 62^\circ\end{aligned}$$

Because angles ACB, ACE, and ECF form a straight angle (180°), we can find the measure of angle ECF.

$$\begin{aligned}m\angle ECF &= 180^\circ - m\angle ACB - m\angle ACE \\ m\angle ECF &= 180^\circ - 62^\circ - 90^\circ \\ m\angle ECF &= 28^\circ\end{aligned}$$

Angles DEC and CEF also form a straight angle.

$$\begin{aligned}m\angle CEF &= 180^\circ - m\angle DEC \\ &= 180^\circ - 59.5^\circ \\ &= 120.5^\circ\end{aligned}$$

The sum of a triangle's interior angles equals 180° , so we can find the measure of angle CFE.

$$\begin{aligned}m\angle CFE &= 180^\circ - m\angle CEF - m\angle ECF \\ &= 180^\circ - 120.5^\circ - 28^\circ \\ &= 31.5^\circ\end{aligned}$$

7. Calculate the lengths of segments $\overline{EF}$ and $\overline{EG}$

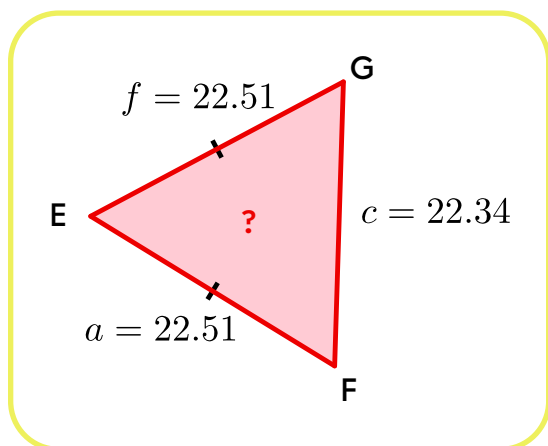
We can use the sine law in triangle CEF.

$$\begin{aligned}\frac{m\overline{EF}}{\sin(\angle ECF)} &= \frac{m\overline{CE}}{\sin(\angle CFE)} \\ \frac{m\overline{EF}}{\sin(28^\circ)} &= \frac{25.05}{\sin(31.5^\circ)} \\ m\overline{EF} &= \frac{25.05 \times \sin(28^\circ)}{\sin(31.5^\circ)} \\ &\approx 22.51\end{aligned}$$

The figure shows that segments $\overline{EF}$ and $\overline{EG}$ are congruent, so segment $\overline{EG}$ also measures 22.51 units.

8. Calculate the area of triangle GEF

Because all 3 sides of triangle GEF are given, we can use Heron's formula.



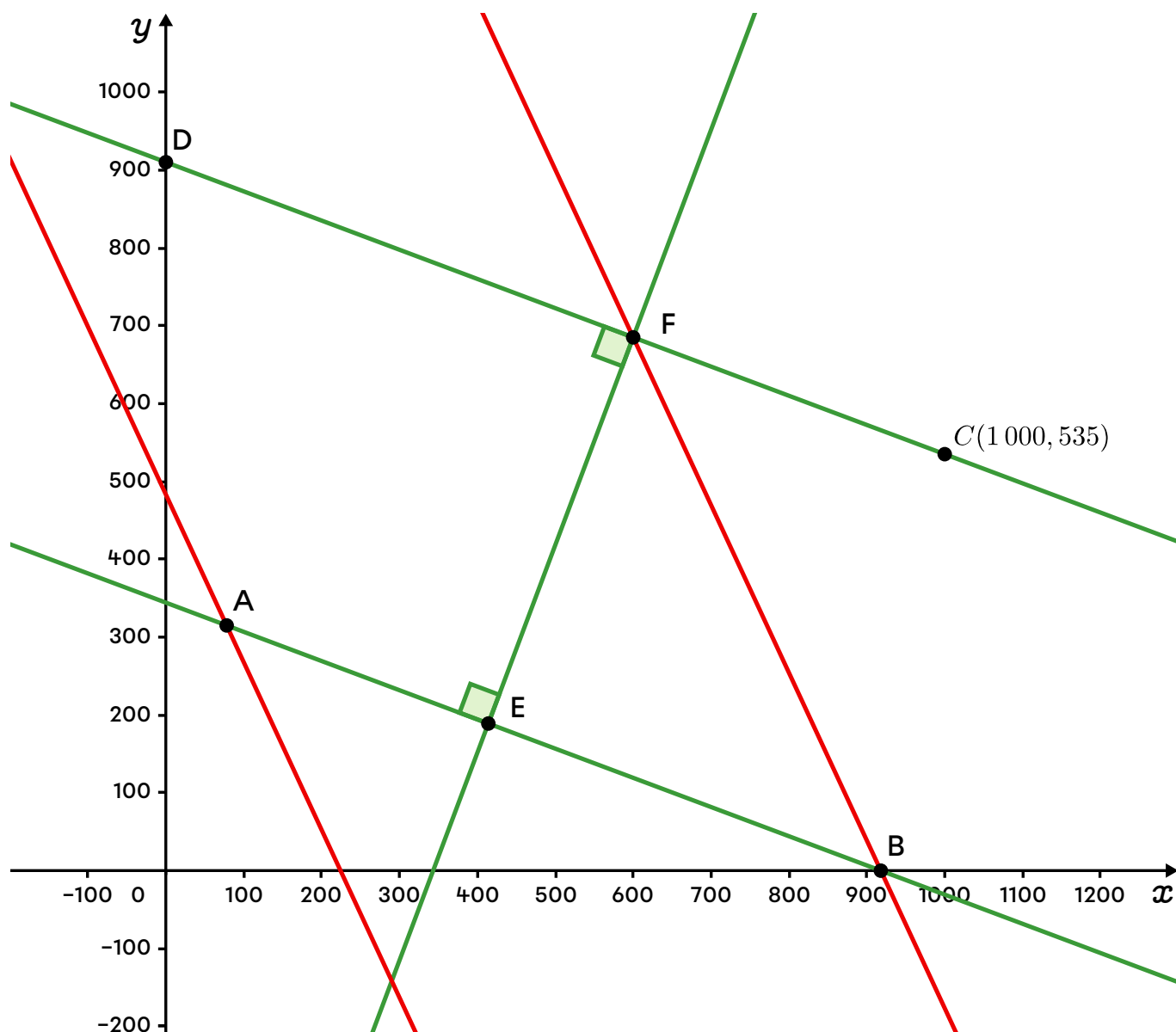
First, we calculate the triangle's half-perimeter (p).

$$\begin{aligned}p &= \frac{a + b + c}{2} \\ &= \frac{22.51 + 22.51 + 22.34}{2} \\ &= 33.68\end{aligned}$$

$$\begin{aligned}A &= \sqrt{p(p-a)(p-b)(p-c)} \\ &= \sqrt{33.68(33.68 - 22.51)(33.68 - 22.51)(33.68 - 22.34)} \\ &\approx 218.3\end{aligned}$$

Answer: Triangle GEF has an area of approximately **218.3** square units.

Question 6



There are two routes from point B to point D: route BFD and route BEFD. To determine the fastest route, calculate the length and duration of each trip, taking speed limits into account. To do this, first determine the coordinates of each point on the graph.

1. Find the coordinates of the points

a) Find the coordinates of point A

In the rule for line AB, replace x with 78 and isolate y .

$$\begin{aligned} 3x + 8y &= 2\,754 \\ 3(78) + 8y &= 2\,754 \\ 234 + 8y &= 2\,754 \\ 8y &= 2\,520 \\ y &= 315 \end{aligned}$$

The coordinates of point A are (78, 315).

b) Find the coordinates of point B

Point B lies on the x -axis, so its y -coordinate is 0.

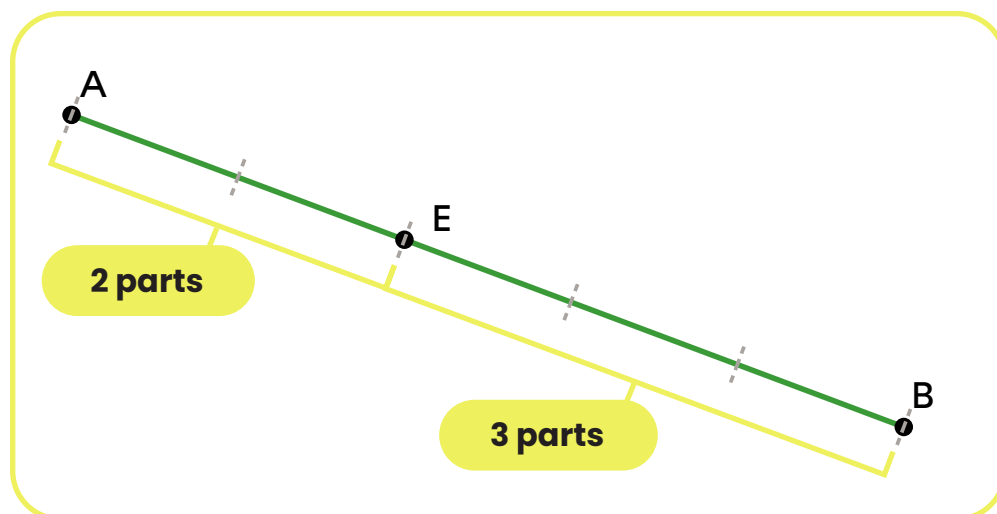
$$\begin{aligned} 3x + 8y &= 2\,754 \\ 3x + 8(0) &= 2\,754 \\ 3x &= 2\,754 \\ x &= 918 \end{aligned}$$

Therefore, the coordinates of point B are (918, 0).

c) Find the coordinates of point E.

Now we use the division point formula.

First, we need to determine what kind of ratio applies to segment $\overline{BA}$. The given ratio (3:2) is a part-to-part ratio. If there are 3 "parts" from point B to point E and another 2 "parts" from point E to point A, then there are 5 "parts" in total.



Therefore, since the starting point is B, we must use the fraction $\frac{3}{5}$.
(If point A is chosen as the starting point, then we must use the fraction $\frac{2}{5}$).

$$\begin{aligned}(x_E, y_E) &= \left(x_B + \frac{3}{5}(x_A - x_B), y_B + \frac{3}{5}(y_A - y_B) \right) \\ &= \left(918 + \frac{3}{5}(78 - 918), 0 + \frac{3}{5}(315 - 0) \right) \\ &= (414, 189)\end{aligned}$$

Therefore, the coordinates of point E are (414, 189).

d) Determine the slope of line AB

The rule for line AB is as follows: $y = -\frac{3}{8}x + 344.25$. Since it is written in the form $y = ax + b$, the slope, or rate of change, corresponds to the coefficient a placed in front of the variable x .

Therefore, the slope of line AB is $-\frac{3}{8}$.

e) Calculate the rule for line EF.

Lines EF and AB are perpendicular, so the slope of EF is the negative reciprocal of the slope of AB. In other words, the product of their slopes would be -1 .

$$\begin{aligned}a_{AB} \times a_{EF} &= -1 \\ a_{EF} &= \frac{-1}{a_{AB}} \\ a_{EF} &= \frac{-1}{-\frac{3}{8}} \\ a_{EF} &= \frac{8}{3}\end{aligned}$$

Note: Another possible explanation is the following. Since lines EF and AB are perpendicular, they have slopes that have opposite signs and are reciprocal fractions. $-\frac{3}{8} \rightarrow +\frac{8}{3}$

Using the slope of EF and the coordinates of point E, we can find the rule for line EF.

$$y = ax + b$$

$$y = \frac{8}{3}x + b$$

$$189 = \frac{8}{3}(414) + b$$

$$189 = 1104 + b$$

$$-915 = b$$

The rule for line EF is $y = \frac{8}{3}x - 915$.

f) Determine the rule for line CD and the coordinates of point D

Because lines AB and CD are both perpendicular to line EF, they are parallel to each other. This also means that AB and CD have the same slope.

$$a_{AB} = a_{CD} = -\frac{3}{8}$$

Using the slope of line CD and the coordinates of point C, we can find the rule for line CD.

$$y = ax + b$$

$$y = -\frac{3}{8}x + b$$

$$535 = -\frac{3}{8}(1000) + b$$

$$535 = -375 + b$$

$$910 = b$$

The rule for line CD is $y = -\frac{3}{8}x + 910$.

→ Because point D lies on the y-axis, the y-intercept of line CD corresponds to the y-coordinate of point D. Thus, the coordinates of point D are (0, 910).

g) Find the coordinates of point F.

Point F lies at the intersection of lines CD and EF. To find the coordinates of this intersection point, we can use the comparison method.

$$\begin{aligned}
 y_{CD} &= y_{EF} \\
 -\frac{3}{8}x + 910 &= \frac{8}{3}x - 915 \\
 910 + 915 &= \frac{8}{3}x + \frac{3}{8}x \\
 1\,825 &= \frac{73}{24}x \\
 600 &= x
 \end{aligned}$$

Replace x with 600 in the rule for line CD to find the y-coordinate of point F.

$$\begin{aligned}
 y &= -\frac{3}{8}x + 910 \\
 y &= -\frac{3}{8}(\mathbf{600}) + 910 \\
 y &= 685
 \end{aligned}$$

The coordinates of point F are (600, 685).

Note: It is also possible to replace x with 600 in the rule for line EF.

2. Calculate the lengths of segments BE, BF, EF, and DF

We can use the formula for finding the distance between 2 points.

$\text{dist}(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	
$\text{dist}(B, E) = \sqrt{(414 - 918)^2 + (189 - 0)^2}$ $m\overline{BE} \approx 538.27 \text{ m}$	$\text{dist}(E, F) = \sqrt{(600 - 414)^2 + (685 - 189)^2}$ $m\overline{EF} \approx 529.73 \text{ m}$
$\text{dist}(B, F) = \sqrt{(600 - 918)^2 + (685 - 0)^2}$ $m\overline{BF} \approx 755.21 \text{ m}$	$\text{dist}(D, F) = \sqrt{(600 - 0)^2 + (685 - 910)^2}$ $m\overline{DF} \approx 640.80 \text{ m}$

3. Calculate the duration of each trip

To calculate the duration of each trip, use the speed formula provided.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \iff \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Possible routes include $B \rightarrow F \rightarrow D$ and $B \rightarrow E \rightarrow F \rightarrow D$.

- a) For route $B \rightarrow F \rightarrow D$, remember that segment BF is limited to 30 km/h, while segment FD is limited to 50 km/h. The distances calculated above were in metres (m), so convert them into kilometres (km).

$$\begin{aligned} \text{Time} &= \frac{\text{distance}(B, F)}{\text{speed}(B, F)} + \frac{\text{distance}(F, D)}{\text{speed}(F, D)} \\ &= \frac{0.75521 \text{ km}}{30 \text{ km/h}} + \frac{0.64080 \text{ km}}{50 \text{ km/h}} \\ &\approx 0.038 \text{ h} \\ &\approx 136.76 \text{ s} \end{aligned}$$

- b) For route $B \rightarrow E \rightarrow F \rightarrow D$, speed is limited to 50 km/h.

$$\begin{aligned} \text{Time} &= \frac{\text{Total distance}}{\text{Speed}} \\ &= \frac{(0.53827 + 0.52973 + 0.64080) \text{ km}}{50 \text{ km/h}} \\ &= \frac{1.70880 \text{ km}}{50 \text{ km/h}} \\ &\approx 0.034 \text{ h} \\ &\approx 123.03 \text{ s} \end{aligned}$$

Conclusion: While route $B \rightarrow E \rightarrow F \rightarrow D$ is longer (in distance) than route $B \rightarrow F \rightarrow D$, it takes approximately 13.73 s less to complete.

Note: For more information on conjectures and proofs, consult the following pages.



Proofs



Conjectures



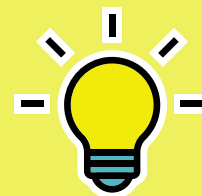
Question 7

Hint: The instructions could be reworded as follows: Using parameters a and b of function $f(x)$, how can we find the rule for function $g(x)$?

When formulating a conjecture, you should analyze at least 3 cases. Typically, we analyze the first case provided, then invent 2 more.

Tips

Work with a variety of examples. Choose whole numbers, decimal numbers, fractions, positive numbers, negative numbers, and so on. For example, if you use only positive integers, you risk not formulating a sufficiently precise conjecture.

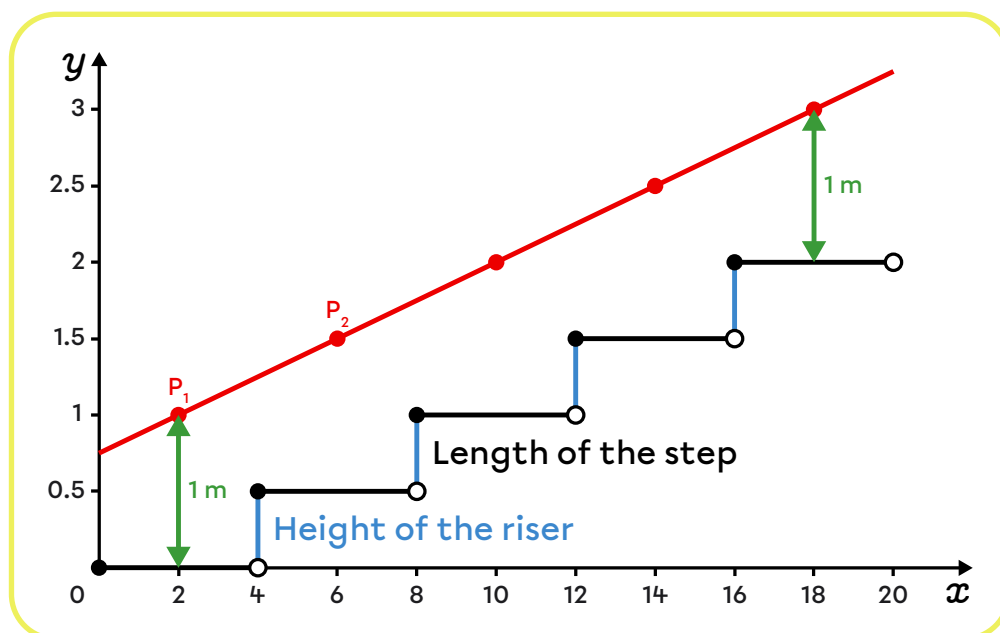


1st case: The staircase ascends, the length is an integer, and the height is a decimal number.

2nd case: The staircase ascends, and the length and height are integers.

3rd case: The staircase descends, and the length and height are decimal numbers.

1st case: The staircase ascends, the length is an integer, and the height is a decimal number



- Determine the staircase dimensions from the graph
 - The height of each riser is 0.5 m.
 - The length of each step is 4 m.

2. Determine the slope of the handrail

a) To determine the handrail slope, we need 2 points on the line.

→ Point P_1 is 1 m above the midpoint of the 1st step, which runs from 0 to 4, so the coordinates of P_1 are (2, 1).

→ Point P_2 is 1 m above the midpoint of the 2nd step, which runs from 4 to 8 at a height of 0.5 m. Therefore, the coordinates of P_2 are (6, 1.5).

b) Now, we calculate the slope (a).

$$\begin{aligned} a &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1.5 - 1}{6 - 2} \\ &= \frac{1}{8} \end{aligned}$$

2nd case: The staircase ascends, and the length and height are integers

1. Select the staircase dimensions and draw a graph

Let's opt for an **ascending** staircase with the following values:

→ Each riser will be 1 m high.

→ Each step will be 3 m long.

2. Determine the slope of the handrail

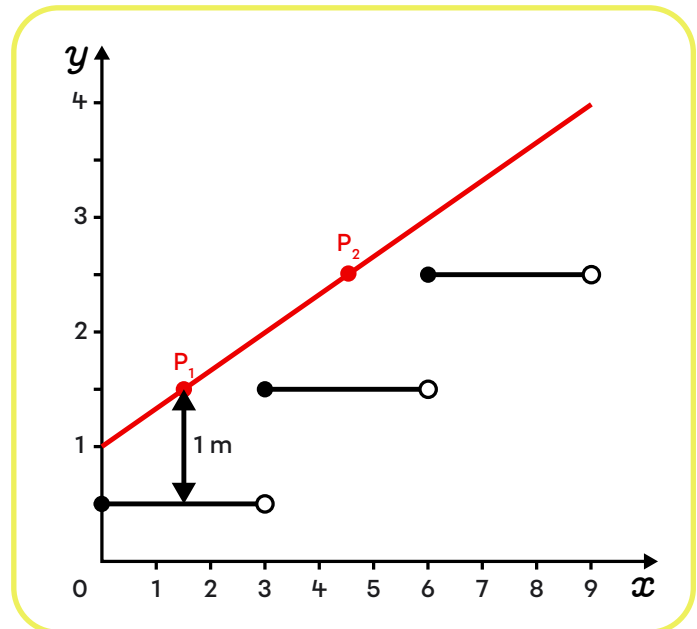
a) To determine the handrail slope, we need 2 points on the line.

→ Point P_1 is 1 m above the midpoint of the 1st step, which runs from 0 to 3 at a height of 0.5 m. Therefore, the coordinates of P_1 are (1.5, 1.5).

→ Point P_2 is 1 m above the midpoint of the 2nd step, which runs from 3 to 6 at a height of 1.5 m. Therefore, the coordinates of P_2 are (4.5, 2.5).

b) Calculate the slope (a).

$$\begin{aligned} a &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2.5 - 1.5}{4.5 - 1.5} \\ &= \frac{1}{3} \end{aligned}$$



3rd case: The staircase descends, and the length and height are decimal numbers

1. Select the staircase dimensions and draw a graph

Let's opt for a **descending** staircase with the following values:

- Each riser will be 0.6 m high.
- Each step will be 0.2 m long.

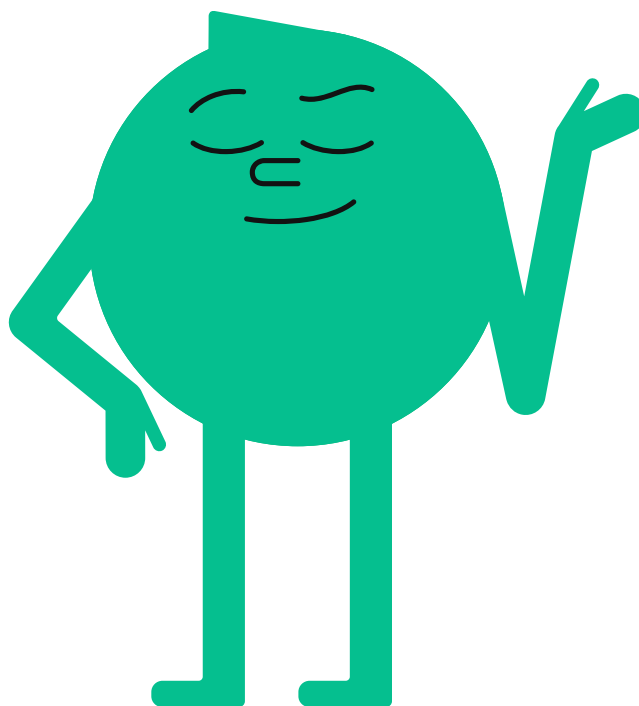
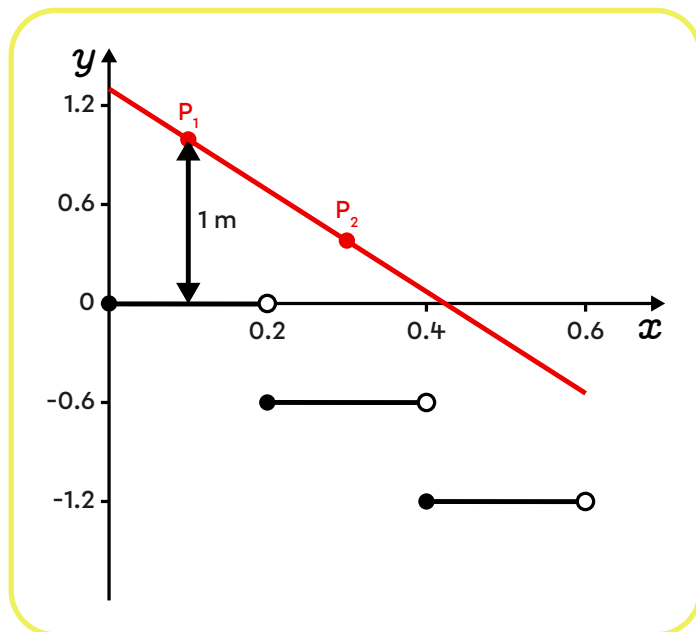
2. Determine the slope of the handrail

- a) To determine the handrail slope, we need 2 points on the line.
 → Point P_1 is 1 m above the midpoint of the 1st step, which runs from 0 to 0.2 at a height of 0 m. Therefore, the coordinates of P_1 are (0.1, 1).

- Point P_2 is 1 m above the midpoint of the 2nd step, which runs from 0.2 to 0.4 at a height of -0.6 m. Therefore, the coordinates of P_2 are (0.3, 0.4).

- b) Let's calculate the slope (a).

$$\begin{aligned}
 a &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{0.4 - 1}{0.3 - 0.1} \\
 &= -3
 \end{aligned}$$



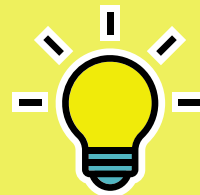
Looking for relationships

Tips

It's often best to work with fractions to see relationships more clearly.

For example, it is easier to see that

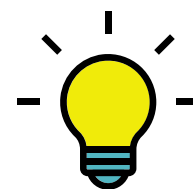
$\frac{1}{2} \div 4 = \frac{1}{8}$ than to see that $0.5 \div 4 = 0.125$.



1st case	Slope = $\frac{1}{8}$	Height = $0.5 = \frac{1}{2}$ Length = 4
2nd case	Slope = $\frac{1}{3}$	Height = 1 Length = 3
3rd case	Slope = -3	Height = $0.6 = \frac{3}{5}$ Length = $0.2 = \frac{1}{5}$

When we're looking for relationships, we often need to ask ourselves questions like the following:

- How can you get $\frac{1}{8}$ using $\frac{1}{2}$ and 4?
- How can you get $\frac{1}{3}$ using 1 and 3?
- How can you get -3 using $-\frac{3}{5}$ and $\frac{1}{5}$?



Tips

- Try basic operations first (addition, subtraction, multiplication, and division).

Examples

$$\frac{1}{2} + 4 \stackrel{?}{=} \frac{1}{8} \quad \text{⊗}$$

$$\frac{1}{2} - 4 \stackrel{?}{=} \frac{1}{8} \quad \text{⊗}$$

$$\frac{1}{2} \times 4 \stackrel{?}{=} \frac{1}{8} \quad \text{⊗}$$

$$\frac{1}{2} \div 4 \stackrel{?}{=} \frac{1}{8} \quad \text{⊙}$$

Verify that **the relationship you've found** also works in the other 2 cases.

- Analyze the graph and use your mathematical knowledge.

Examples

◦ The slope corresponds to $\frac{\Delta y}{\Delta x}$.

- The height of each riser corresponds to Δy .
- The length of each step corresponds to Δx .
- The line passing through the points at the beginning of each step is parallel to the handrail. Therefore, they have the same slope.
- The slope of the 2 lines can therefore be found by performing the following calculation.

$$\frac{\Delta y}{\Delta x} = \frac{\text{Height of a riser}}{\text{Length of a step}}$$

- Pay close attention to the positive and negative signs (+ and -).

Examples

◦ 1st case: $\frac{1}{2} \div 4 \stackrel{?}{=} \frac{1}{8} \quad \text{⊙}$

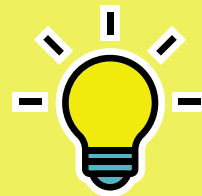
◦ 2nd case: $1 \div 3 = \frac{1}{3} \quad \text{⊙}$

◦ 3rd case: $\frac{3}{5} \div \frac{1}{5} = 3 \quad \text{⊗} \quad \text{The slope must be -3.}$

We're now ready to formulate a complete answer.

Tips

- You must write your answer, i.e., your conjecture, using complete sentences. In other words, while " $a = \text{Height} \div \text{Length}$ " can be included in your conjecture, this is not the expected answer.
- Multiple different formulations are possible.
- It's better to respond with a relationship that doesn't work in every case than none at all.



Answer

The handrail slope (a) is equal to the ratio between the height of the risers and the length of the steps when the staircase is ascending.

If the staircase is descending, then the slope is equal to the opposite of this same ratio.

Note: Your final conjecture must account for the change in sign when the staircase is descending, otherwise it will be considered incomplete.

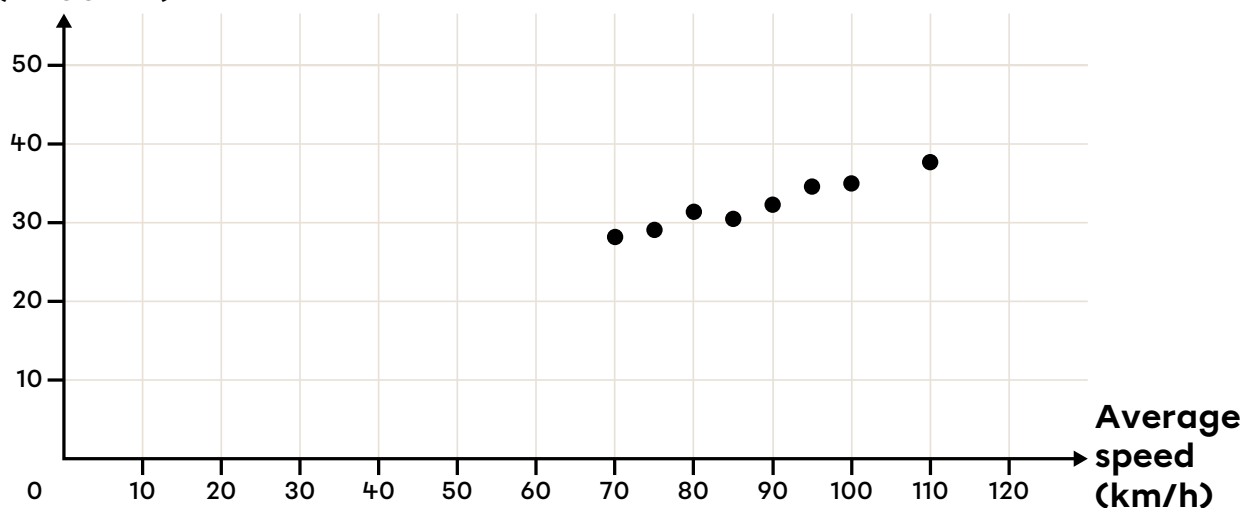
Question 8

The main steps in solving the problem are as follows: We need to determine the regression line equation for the MetaCargo fleet data, then find the rule for fuel consumption as a function of average speed for the NovaFret fleet. Next, we need to use various speeds to analyze which truck fleet consumes less fuel.

A. Let's start with the **MetaCargo** fleet.

We create a scatter plot to see if there's a correlation between the variables.

Fuel consumption
(L/100 km)



There is a positive correlation between the variables, meaning the higher a truck's average speed, the higher its fuel consumption.

So, we're going to find the regression line rule. We can use either the **Mayer method** or median-median method. Let's use the Mayer method.

1. Order coordinates according to the independent variable.
This has already been done.

x: Average speed (km/h)	70	75	80	85	90	95	100	110
y: Fuel consumption (L/100 km)	28.2	29.1	31.4	30.5	32.3	34.6	35.0	37.7

2. Separate the distribution into 2 equal groups if possible.

	1st group				2nd group			
x: Average speed (km/h)	70	75	80	85	90	95	100	110
y: Fuel consumption (L/100 km)	28.2	29.1	31.4	30.5	32.3	34.6	35.0	37.7

3. Calculate the **average point** for each group (P_1 and P_2).

	1st group	2nd group
Average x-coordinate	$\bar{x}_1 = \frac{70 + 75 + 80 + 85}{4} = 77.5$	$\bar{x}_2 = \frac{90 + 95 + 100 + 110}{4} = 98.75$
Average y-coordinate	$\bar{y}_1 = \frac{28.2 + 29.1 + 31.4 + 30.5}{4} = 29.8$	$\bar{y}_2 = \frac{32.3 + 34.6 + 35.0 + 37.7}{4} = 34.9$
Average point	$P_1 (77.5, 29.8)$	$P_2 (98.75, 34.9)$

4. Find the rule of the regression line ($y = ax + b$) passing through points P_1 and P_2 .

a) Calculate the slope.

$$\begin{aligned}
 a &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{34.9 - 29.8}{98.75 - 77.5} \\
 &= \frac{5.1}{21.25} \\
 &= 0.24
 \end{aligned}$$

Therefore, $y = 0.24x + b$.

b) Calculate the value of b using point P_1 (77.5, 29.8).

$$\begin{aligned} y &= 0.24x + b \\ 29.8 &= 0.24(77.5) + b \\ 11.2 &= b \end{aligned}$$

Thus, the regression line rule for MetaCargo is $y_1 = 0.24x + 11.2$.

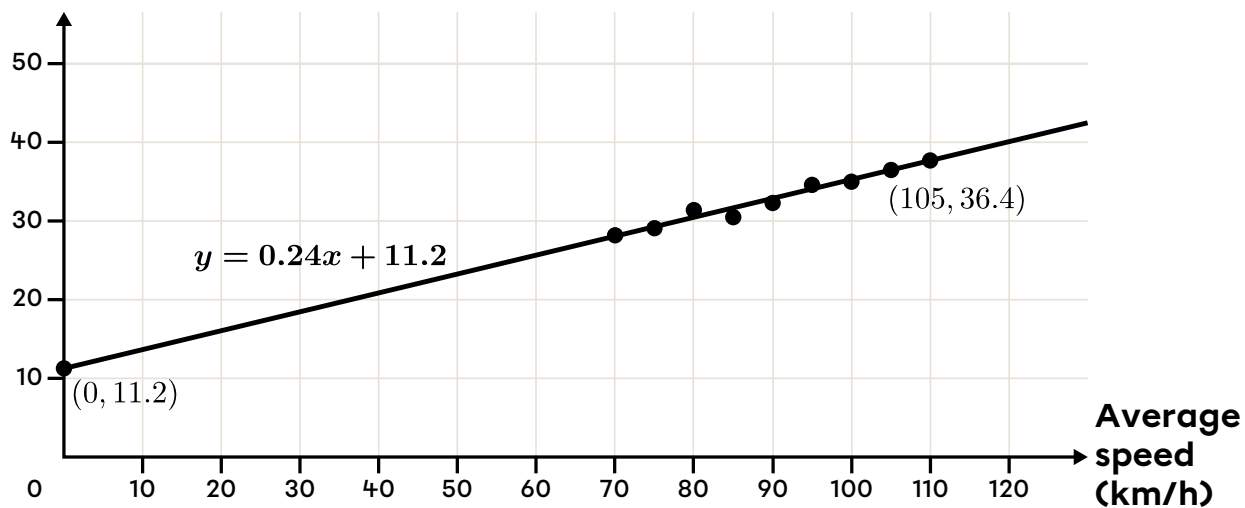
Note: If you used the median-median method, you should get $y_1 = 0.236x + 11.18\bar{3}$.

5. Predict fuel consumption at 105 km/h using the line rule.

To predict fuel consumption for a truck driving at an average speed of 105 km/h, we need to replace x by 105 in the regression line.

$$\begin{aligned} y_1 &= 0.24x + 11.2 \\ &= 0.24(105) + 11.2 \\ &= 36.4 \text{ L/100 km} \end{aligned}$$

Fuel consumption
(L/100 km)



B. NovaFret.

- Find the rule for the function that allows us to calculate fuel consumption for **NovaFret** trucks.

- We have the model $y = a(c)^x$.
- We have the y-intercept, which is the point $(0, 13.8)$.
The y-intercept corresponds to parameter a . Therefore, $a = 13.8$.
- We know that fuel consumption increases 1% per km/h.
An increase of 1% means that the base c equals 1.01, because $100\% + 1\% = 101\% = 1.01$.

Therefore, the rule for the function that gives us fuel consumption ($C(x)$) relative to average speed (x) is as follows.

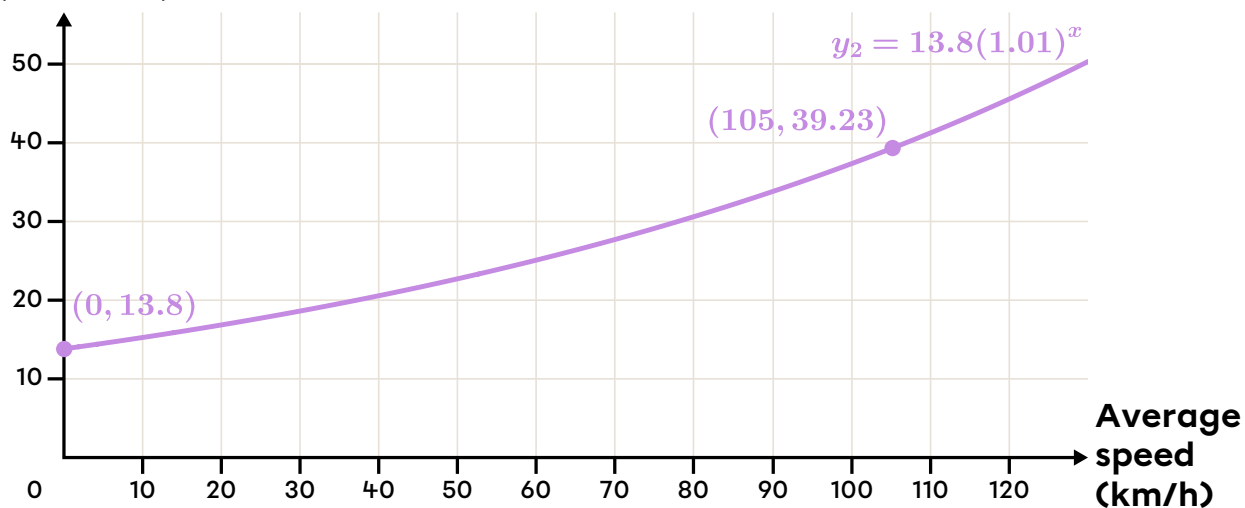
$$y_2 = 13.8(1.01)^x$$

- Calculate fuel consumption at 105 km/h.

Replace x by 105 in the rule.

$$\begin{aligned} y_2 &= 13.8(1.01)^x \\ &= 13.8(1.01)^{105} \\ &\approx 39.23 \text{ L/100 km} \end{aligned}$$

Fuel consumption
(L/100 km)



Answer: Given that the average route speed is 105 km/h, he should use trucks in the **MétaCargo** fleet.

Justification: According to the regression line, fuel consumption for the trucks in the MetaCargo fleet at 105 km/h is approximately 36.4 L/100 km, which is lower than fuel consumption for the trucks in the NovaFret fleet, which is around 39.23 L/100 km.

**Fuel consumption
(L/100 km)**

