Ministry Exam Simulation

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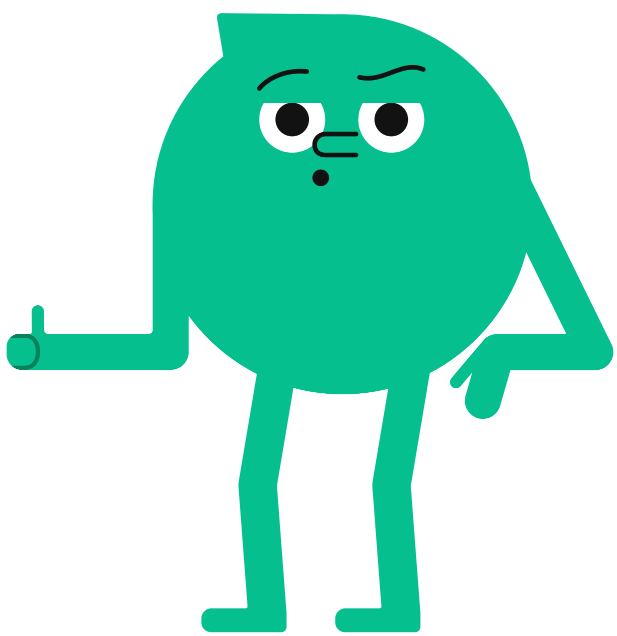
Mathematics TS
Secondary 4

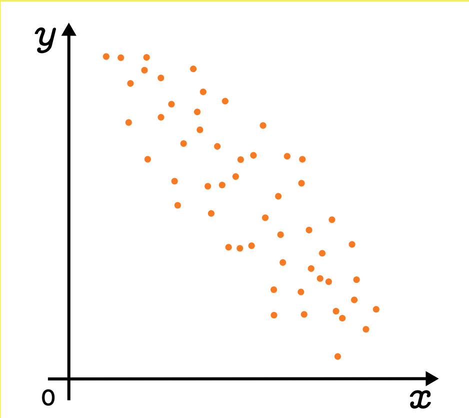
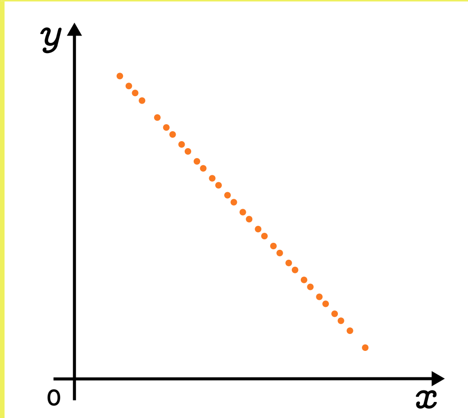
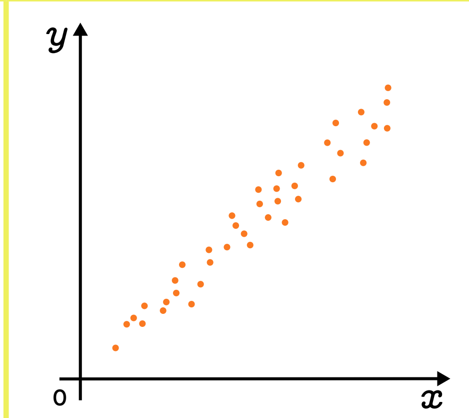


Answer Key

Section A

Question 1

Place the following scatter plots in **descending** order of linear correlation **coefficient**.

A	B	C
		

a) B > C > A
b) A < C < B
c) C > A > B
d) B < A < C

Answer: c) C > A > B

Explanations

Scatter plot **C** is the only graph displaying an **upward** trend. It is therefore the only one with a **positive** correlation coefficient. Because the remaining 2 are negative, C must necessarily have the highest correlation coefficient.

Tip: Because they should be arranged in *descending order*, the correct answer must begin with **C**. Therefore, **by process of elimination**, choices a), b), and d) can be ruled out.

Let's go into more detail. We'll start by estimating the correlation coefficients of the 3 scatter plots.

- Graph **A** shows a **weak negative** correlation. Therefore, $\mathbf{r_A \approx -0.6}$.
- Graph **B** shows a **perfect negative** correlation. Therefore, $\mathbf{r_B = -1}$.
- Graph **C** shows a **moderate positive** correlation. Therefore, $\mathbf{r_C \approx +0.8}$.

Thus, in *descending order*, we get +0.8 > −0.6 > −1, i.e., answer **c) C > A > B**.
Answer **d) B < A < C** is incorrect because it appears in *ascending* rather than *descending order*.

If we had been asked to list the scatter plots in order of correlation **strength**, we would have ignored the + and - signs.

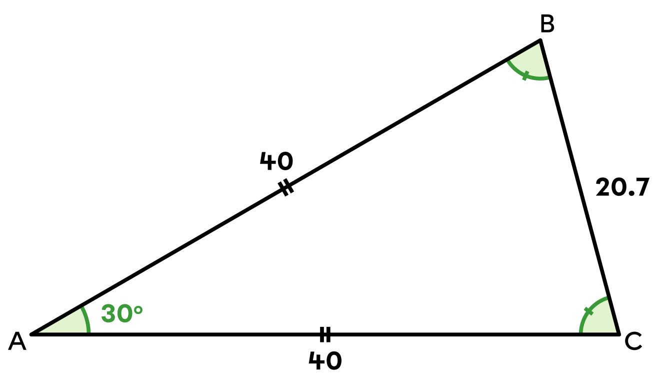
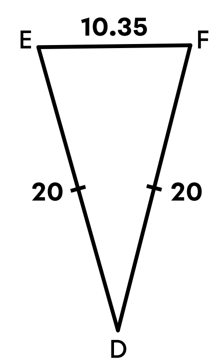
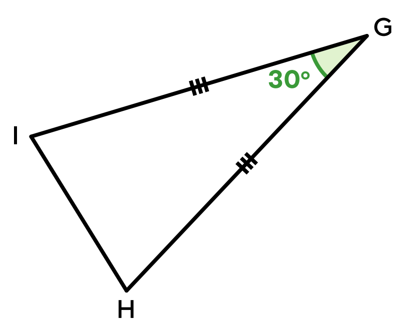
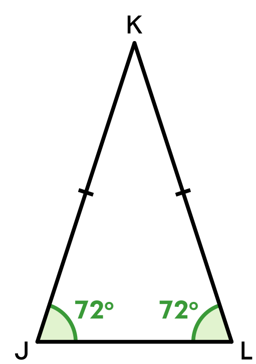
- The scatter plot with the weakest correlation is graph A.
- Scatter plot C has a stronger correlation than scatter plot A.
- Scatter plot B has an even stronger correlation because it is perfect.

Thus, from weakest to strongest correlation (regardless of signs), the order is **b) A < C < B**. From strongest to weakest correlation, the order is reversed, i.e., option **a) B > C > A**.

Keep in mind: During the exam, be sure to pay attention to the order of the information (ascending or descending) and what must be put in order, e.g., the correlation **coefficients** (take the plus and minus signs into account) or correlation **strength** (ignore the signs).

Question 2

Which of these doesn't belong?
All but one of the following triangles are similar. Which one?

A	B
	
C	**D**
	

Answer: d) triangle JKL

- Triangle ABC is similar to triangle DEF by **SSS**: Triangles are similar if their corresponding sides are proportional.

$$\begin{array}{ccccc}\dfrac{\text{m}\overline{AB}}{\text{m}\overline{DE}} & = & \dfrac{\text{m}\overline{AC}}{\text{m}\overline{DF}} & = & \dfrac{\text{m}\overline{BC}}{\text{m}\overline{EF}}\\ \dfrac{40}{20} & = & \dfrac{40}{20} & = & \dfrac{20.7}{10.35}\\ 2 & = & 2 & = & 2\end{array}$$

- Triangle ABC is similar to triangle GHI by **SAS**: Triangles are similar if they have 1 pair of congruent angles between 2 pairs of proportional sides.

  - $\text{m}\angle BAC = \text{m}\angle HGI = 30^\circ$
  - $$\begin{array}{ccc}\dfrac{\text{m}\overline{AB}}{\text{m}\overline{GH}} & = & \dfrac{\text{m}\overline{AC}}{\text{m}\overline{GI}}\\ \dfrac{40}{\text{m}\overline{GH}} & = & \dfrac{40}{\text{m}\overline{GI}}\end{array}$$

  This equality is true because segments GH and GI are congruent.

  Note: In these 2 triangles, the congruent angles are indeed located between the proportional sides.

Since triangle ABC is similar to triangle DEF **and** triangle GHI, we can assert, via the transitive property, that triangles DEF and GHI are similar to each other. In other words, we've already identified the 3 similar triangles. By process of elimination, the remaining triangle, KJL, doesn't belong. Here's why.

- Triangles ABC and KJL appear similar by AA (triangles are similar if they have 2 pairs of congruent angles), but they are not. Angles B and C are congruent to each other, as are angles J and L, but this doesn't mean that angle C is congruent to angle J. Let's see why.
  In triangle ABC, angle A measures 30°. Since the sum of a triangle's interior angles equals 180°, we can calculate the measures of angles B and C.

$$\text{m}\angle B = \text{m}\angle C = \frac{180^\circ - 30^\circ}{2} = 75^\circ$$

  In triangle KJL, angles J and L measure 72°, not 75°.

Notes:

1. Congruence conditions for triangles are virtually identical to similarity conditions.
   - For conditions SSS and SAS, sides must be *congruent* rather than *proportional*.
   - Use **ASA** instead of **AA**: Triangles are congruent if they have 1 pair of congruent sides between 2 pairs of congruent corresponding angles.

2. Before referring to the minimal conditions of similarity or congruence, it is sometimes possible to find missing measures of triangles using the cosine and sine laws. However, this is generally not necessary.

Section B

Question 3

Consider a parabola passing through the point (8, 48). What is the x-coordinate, in fractional notation, of the point or points on the parabola whose y-coordinate is $\frac{4}{27}$?

Answer: The point or points with the y-coordinate $\frac{4}{27}$ have the x-coordinate **- 4 / 9 and 4 / 9**.

Explanations

1. Find the rule of the parabola.

   First, we calculate parameter a using point (8, 48).

$$\begin{aligned} f(x) &= ax^2 \\ 48 &= a(8)^2 \\ 48 &= 64a \\ \frac{48}{64} &= a \\ \frac{3}{4} &= a \end{aligned}$$

   This gives us the following rule:

$$f(x) = \frac{3}{4}x^2$$

2. Calculate the x-coordinates when the y-coordinate is $\frac{4}{27}$.

   We replace *f(x)* by $\frac{4}{27}$, then isolate *x*.

$$\begin{aligned} f(x) &= \frac{3}{4}x^2 \\ \frac{4}{27} &= \frac{3}{4}x^2 \\ \frac{4}{27} \times \frac{4}{3} &= x^2 \\ \frac{16}{81} &= x^2 \\ \pm\sqrt{\frac{16}{81}} &= x \\ \pm\frac{4}{9} &= x \end{aligned}$$

This gives us the points $(\frac{-4}{9}, \frac{4}{27})$ and $(\frac{4}{9}, \frac{4}{27})$.

Question 4

The area of a quadrilateral is represented by the following algebraic expression:

$$25x^2 - 35x + 12$$

Identify the type of quadrilateral and the binomials with integer coefficients corresponding to the lengths of its sides.

Answer: The sides of this **rectangle** are represented by the expressions **5x - 4** and **5x - 3**.

Explanations

To find the expressions corresponding to the sides of the quadrilateral, we need to factor the polynomial $25x^2-35x+12$. Several factoring methods are possible. Let's use the *product-sum* technique.

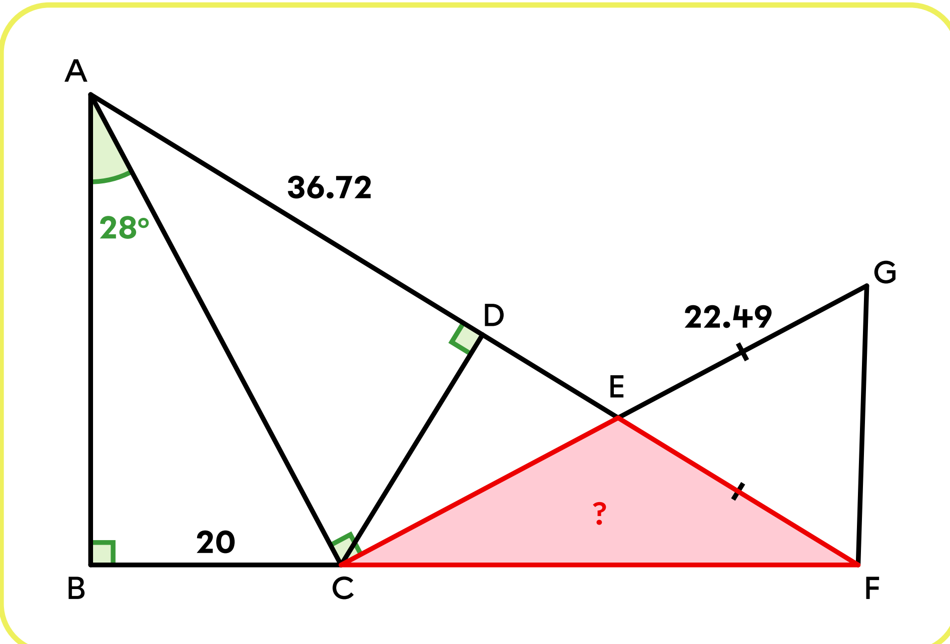
Product-sum
$25x^2 - 35x + 12$ $\text{Product} = 25 \times 12 \quad \text{Sum} = -35$ $= 300$ We'll use -20 and -15. $25x^2 - 20x - 15x + 12$ $5x(5x-4) - 3(5x-4)$ $(5x-4)(5x-3)$

Because the 2 expressions we get differ for all x values, we can conclude that the quadrilateral is a **rectangle** whose sides are 5x - 4 and 5x - 3.

Section C

Question 5

What is the area of triangle CEF?



Several procedures are possible. Here's one.

1. Calculate the length of segment $\overline{\text{AC}}$

$$\sin(A) = \frac{\text{m}\overline{BC}}{\text{m}\overline{AC}}$$
$$\sin(28^\circ) = \frac{20}{\text{m}\overline{AC}}$$
$$\text{m}\overline{AC} = \frac{20}{\sin(28^\circ)}$$
$$\text{m}\overline{AC} \approx 42.6$$

2. Calculate the length of segment $\overline{\text{AE}}$

   Each leg ($\overline{\text{AC}}$) of a right triangle (ΔACE) is the proportional mean between its projection onto the hypotenuse ($\overline{\text{AD}}$) and the entire hypotenuse ($\overline{\text{AE}}$).

$$\frac{\text{m}\overline{AD}}{\text{m}\overline{AC}} = \frac{\text{m}\overline{AC}}{\text{m}\overline{AE}}$$
$$\text{m}\overline{AC}^2 = \text{m}\overline{AD} \times \text{m}\overline{AE}$$
$$42.6^2 = 36.72 \times \text{m}\overline{AE}$$
$$49.42 \approx \text{m}\overline{AE}$$

3. Calculate the length of segment $\overline{\mathrm{DE}}$.

   Points A, D, and E are aligned. Therefore, $\mathrm{m}\overline{\mathrm{AE}} = \mathrm{m}\overline{\mathrm{AD}} + \mathrm{m}\overline{\mathrm{DE}}$.

$$\mathrm{m}\overline{AE} = \mathrm{m}\overline{AD} + \mathrm{m}\overline{DE}$$

$$\begin{aligned}\mathrm{m}\overline{DE} &= \mathrm{m}\overline{AE} - \mathrm{m}\overline{AD}\\ &= 49.42 - 36.72\\ &= 12.7\end{aligned}$$

4. Calculate the length of segment $\overline{\mathrm{CD}}$

   In a right triangle (ΔACE), the height drawn from the right angle ($\overline{\mathrm{CD}}$) is the proportional mean between the 2 segments it creates on the hypotenuse ($\overline{\mathrm{AD}}$ and $\overline{\mathrm{DE}}$).

$$\frac{\mathrm{m}\overline{AD}}{\mathrm{m}\overline{CD}} = \frac{\mathrm{m}\overline{CD}}{\mathrm{m}\overline{DE}}$$

$$\begin{aligned}\mathrm{m}\overline{CD}^2 &= \mathrm{m}\overline{AD} \times \mathrm{m}\overline{DE}\\ \mathrm{m}\overline{CD} &= \sqrt{36.72 \times 12.7}\\ \mathrm{m}\overline{CD} &\approx 21.59\end{aligned}$$

5. Calculate the length of segment $\overline{\mathrm{CE}}$

   In a right triangle (ΔACE), the product of the hypotenuse ($\overline{\mathrm{AE}}$) and the corresponding height ($\overline{\mathrm{CD}}$) is equal to the product of the legs ($\overline{\mathrm{AC}}$ and $\overline{\mathrm{CE}}$).

$$\begin{aligned}\mathrm{m}\overline{AE} \times \mathrm{m}\overline{CD} &= \mathrm{m}\overline{AC} \times \mathrm{m}\overline{CE}\\ \left(\mathrm{m}\overline{AD} + \mathrm{m}\overline{DE}\right) \times \mathrm{m}\overline{CD} &= \mathrm{m}\overline{AC} \times \mathrm{m}\overline{CE}\\ (36.72 + 12.7) \times 21.59 &= 42.6 \times \mathrm{m}\overline{CE}\\ \mathrm{m}\overline{CE} &= \frac{49.42 \times 21.59}{42.6}\\ \mathrm{m}\overline{CE} &\approx 25.05\end{aligned}$$

6. Calculate the measures of angles DEC and CEF.

   Triangle DEC is right-angled at D, so we can use a trigonometric ratio.

$$\begin{aligned}\tan(\angle DEC) &= \frac{\mathrm{m}\overline{CD}}{\mathrm{m}\overline{DE}}\\ &= \frac{21.59}{12.7}\end{aligned}$$

$$\begin{aligned}\mathrm{m}\angle DEC &= \tan^{-1}\left(\frac{21.59}{12.7}\right)\\ &\approx 59.5^\circ\end{aligned}$$

Adjacent angles that form a straight angle are supplementary. Using this rule, we can calculate the measure of angle CEF.

$$\begin{aligned}\mathrm{m}\angle CEF &= 180^\circ - \mathrm{m}\angle DEC\\ &= 180^\circ - 59.5^\circ\\ &= 120.5^\circ\end{aligned}$$

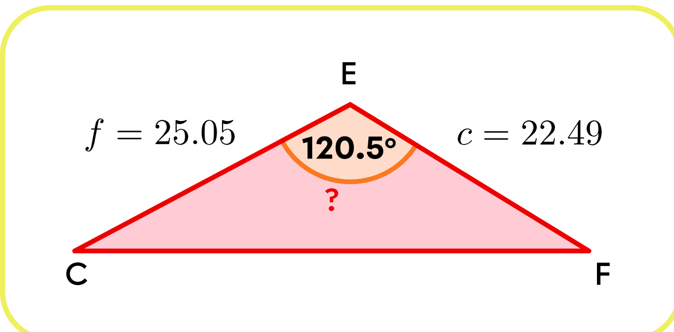
7. Calculate the measure of segment $\overline{\mathrm{EF}}$.

   The markings on segments $\overline{\mathrm{EF}}$ and $\overline{\mathrm{EG}}$ tell us that they are congruent. This is information we can use to solve the problem.

$$\mathrm{m}\overline{EF} = \mathrm{m}\overline{EG} = 22.49 \text{ units}$$

8. Calculate the area of triangle CEF.

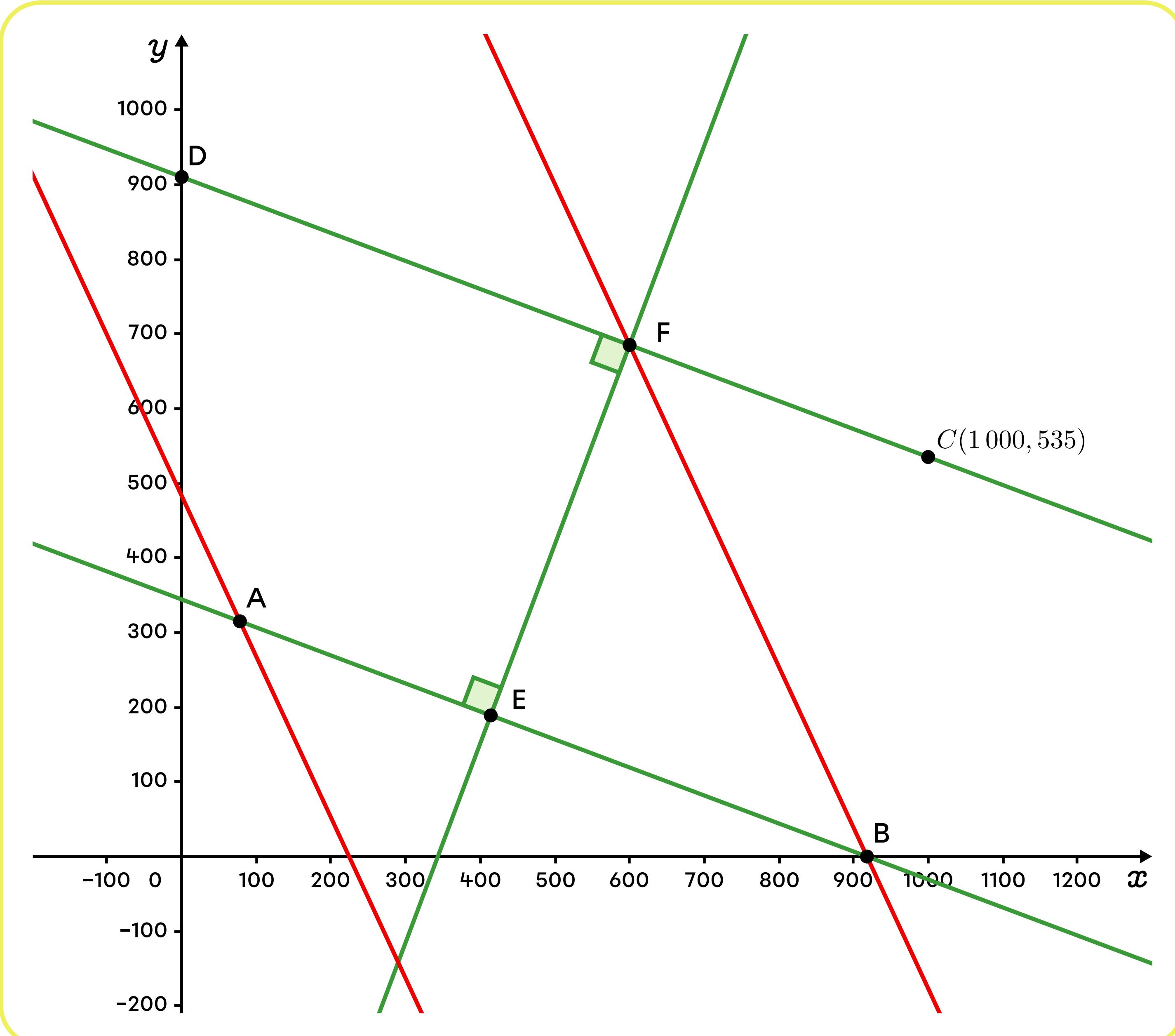
   Now, we can use the area formula along with the sine trigonometric ratio.



$$\begin{aligned}A_{\triangle CEF} &= \frac{c \times f \times \sin(E)}{2}\\ &= \frac{22.49 \times 25.05 \times \sin(120.5^\circ)}{2}\\ &\approx 242.7 \text{ units}^2\end{aligned}$$

Answer: Triangle CEF has an area of approximately 242,7 square units.

Question 6



There are two routes from point B to point D: route BFD and route BEFD. To determine the fastest route, calculate the length and duration of each trip, taking speed limits into account. To do this, first determine the coordinates of each point on the graph.

1. Find the coordinates of the points

a) Find the coordinates of point A

In the rule for line AB, replace x with 78 and isolate y.

$$\begin{aligned} 3x + 8y &= 2\ 754 \\ 3(78) + 8y &= 2\ 754 \\ 234 + 8y &= 2\ 754 \\ 8y &= 2\ 520 \\ y &= 315 \end{aligned}$$

The coordinates of point A are (78, 315).

b) Find the coordinates of point B

Point B lies on the x-axis, so its y-coordinate is 0.

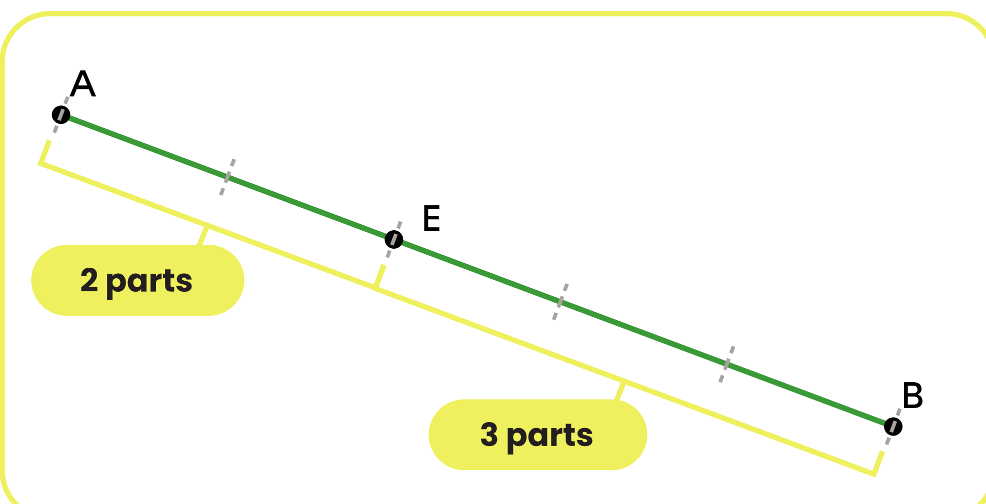
$$\begin{aligned} 3x + 8y &= 2\ 754 \\ 3x + 8(0) &= 2\ 754 \\ 3x &= 2\ 754 \\ x &= 918 \end{aligned}$$

Therefore, the coordinates of point B are (918, 0).

c) Find the coordinates of point E.

Now we use the division point formula.

First, we need to determine what kind of ratio applies to segment $\overline{\text{BA}}$. The given ratio (3:2) is a part-to-part ratio. If there are 3 “parts” from point B to point E and another 2 “parts” from point E to point A, then there are 5 “parts” in total.



Therefore, since the starting point is B, we must use the fraction $\frac{3}{5}$.
(If point A is chosen as the starting point, then we must use the fraction $\frac{2}{5}$).

$$\begin{aligned}(x_E, y_E) &= \left(x_B + \frac{3}{5}(x_A - x_B), y_B + \frac{3}{5}(y_A - y_B)\right)\\ &= \left(918 + \frac{3}{5}(78 - 918), 0 + \frac{3}{5}(315 - 0)\right)\\ &= (414, 189)\end{aligned}$$

Therefore, the coordinates of point E are (414, 189).

d) Determine the slope of line AB

Simply convert the rule for line AB from the general form to the functional form.

$$\begin{aligned}3x + 8y &= 2754\\ 8y &= -3x + 2754\\ y &= -\frac{3}{8}x + 344.25\end{aligned}$$

Therefore, the slope of line AB is - $\frac{3}{8}$.

e) Calculate the rule for line EF.

Lines EF and AB are perpendicular, so the slope of EF is the negative reciprocal of the slope of AB. In other words, the product of their slopes would be -1.

$$\begin{aligned}a_{AB} \times a_{EF} &= -1\\ a_{EF} &= \frac{-1}{a_{AB}}\\ a_{EF} &= \frac{-1}{-\frac{3}{8}}\\ a_{EF} &= \frac{8}{3}\end{aligned}$$

Note: Another possible explanation is the following. Since lines EF and AB are perpendicular, they have slopes that have opposite signs and are reciprocal fractions. - $\frac{3}{8}$ → + $\frac{8}{3}$

Using the slope of EF and the coordinates of point E, we can find the rule for line EF.

$$y = \boldsymbol{a}x + b$$
$$\boldsymbol{y} = \boldsymbol{\frac{8}{3}}\boldsymbol{x} + b$$
$$\mathbf{189} = \frac{8}{3}\mathbf{(414)} + b$$
$$189 = 1\ 104 + b$$
$$-915 = b$$

The rule for line EF is $y = \frac{8}{3}x - 915$.

f) Determine the rule for line CD and the coordinates of point D

Because lines AB and CD are both perpendicular to line EF, they are parallel to each other. This also means that AB and CD have the same slope.

$$a_{AB} = a_{CD} = -\frac{3}{8}$$

Using the slope of line CD and the coordinates of point C, we can find the rule for line CD.

$$y = \boldsymbol{a}x + b$$
$$\boldsymbol{y} = \boldsymbol{-\frac{3}{8}}\boldsymbol{x} + b$$
$$\mathbf{535} = -\frac{3}{8}\mathbf{(1\,000)} + b$$
$$535 = -375 + b$$
$$910 = b$$

The rule for line CD is $y = -\frac{3}{8}x + 910$.

→ Because point D lies on the y-axis, the y-intercept of line CD corresponds to the y-coordinate of point D. Thus, the coordinates of point D are (0, 910).

g) Find the coordinates of point F.

Point F lies at the intersection of lines CD and EF. To find the coordinates of this intersection point, we can use the comparison method.

$$\begin{aligned} y_{CD} &= y_{EF} \\ -\frac{3}{8}x + 910 &= \frac{8}{3}x - 915 \\ 910 + 915 &= \frac{8}{3}x + \frac{3}{8}x \\ 1\ 825 &= \frac{73}{24}x \\ 600 &= x \end{aligned}$$

Replace x with 600 in the rule for line CD to find the y-coordinate of point F.

$$\begin{aligned} y &= -\frac{3}{8}\boldsymbol{x} + 910 \\ y &= -\frac{3}{8}\mathbf{(600)} + 910 \\ y &= 685 \end{aligned}$$

The coordinates of point F are (600, 685).

Note: It is also possible to replace x with 600 in the rule for line EF.

2. Calculate the lengths of segments BE, BF, EF, and DF

We can use the formula for finding the distance between 2 points.

$\text{dist}(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	
$\text{dist}(B, E) = \sqrt{(414 - 918)^2 + (189 - 0)^2}$ $\text{m}\overline{BE} \approx 538.27 \text{ m}$	$\text{dist}(E, F) = \sqrt{(600 - 414)^2 + (685 - 189)^2}$ $\text{m}\overline{EF} \approx 529.73 \text{ m}$
$\text{dist}(B, F) = \sqrt{(600 - 918)^2 + (685 - 0)^2}$ $\text{m}\overline{BF} \approx 755.21 \text{ m}$	$\text{dist}(D, F) = \sqrt{(600 - 0)^2 + (685 - 910)^2}$ $\text{m}\overline{DF} \approx 640.80 \text{ m}$

3. Calculate the duration of each trip

To calculate the duration of each trip, use the speed formula provided.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \iff \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Possible routes include B → F → D and B → E → F → D.

a) For route B → F → D, remember that segment BF is limited to 30 km/h, while segment FD is limited to 50 km/h. The distances calculated above were in metres (m), so convert them into kilometres (km).

$$\begin{aligned}\text{Time} &= \frac{\text{distance}(B,F)}{\text{speed}(B,F)} + \frac{\text{distance}(F,D)}{\text{speed}(F,D)}\\ &= \frac{0.75521\ \text{km}}{30\ \text{km/h}} + \frac{0.64080\ \text{km}}{50\ \text{km/h}}\\ &\approx 0.038\ \text{h}\\ &\approx 136.76\ \text{s}\end{aligned}$$

b) For route B → E → F → D, speed is limited to 50 km/h.

$$\begin{aligned}\text{Time} &= \frac{\text{Total distance}}{\text{Speed}}\\ &= \frac{(0.53827 + 0.52973 + 0.64080)\ \text{km}}{50\ \text{km/h}}\\ &= \frac{1.70880\ \text{km}}{50\ \text{km/h}}\\ &\approx 0.034\ \text{h}\\ &\approx 123.03\ \text{s}\end{aligned}$$

Conclusion: While route B → E → F → D is longer (in distance) than route B → F → D, it takes approximately 13.73 s less to complete.

Note: For more information on conjectures and proofs, consult the following pages.

Proofs

Conjectures

Question 7

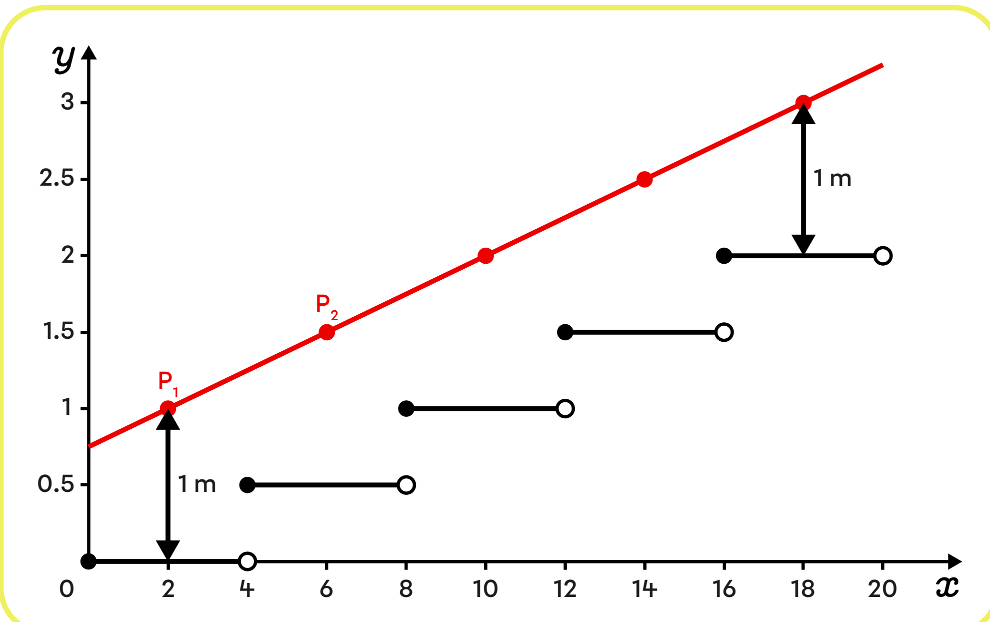
Hint: The instructions could be reworded as follows: Using parameters a and b of function f(x), how can we find the rule for function g(x)?

When formulating a conjecture, you should analyze at least 3 cases. Typically, we analyze the first case provided, then invent 2 more.

Tips
Work with a variety of examples. Choose whole numbers, decimal numbers, fractions, positive numbers, negative numbers, and so on. For example, if you use only positive integers, you risk not formulating a sufficiently precise conjecture.

1st case: Parameters a and b are positive.
2nd case: Parameters a and b are negative.
3rd case: Parameter a is negative and parameter b is positive.

1st Case: Parameters a and b are positive



1. Determine the value of parameters a and b of the staircase.
   → The height of each riser is 0.5 m. Therefore, $|a| = 0.5$.
   → The length of each step is 4 m. Therefore, $|b| = \frac{1}{4} = 0.25$.
   → The staircase ascends, and each step begins with a full point, so a and b are positive.

   In summary, the rule is $f(x) = 0.5[0.25x]$.

2. Determine the rule for the handrail (g(x)).

   a) To determine the handrail rule, we need 2 points on the line.
   → Point P_1 is 1 m above the midpoint of the 1st step, which runs from 0 to 4, so the coordinates of P_1 are (2, 1).
   → Point P_2 is 1 m above the midpoint of the 2nd step, which runs from 4 to 8 at a height of 0.5 m. Therefore, the coordinates of P_2 are (6, 1.5).

   b) Calculate the slope (m).

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1.5 - 1}{6 - 2} \\ &= \frac{1}{8} \end{aligned}$$

   c) Determine the y-intercept (k).

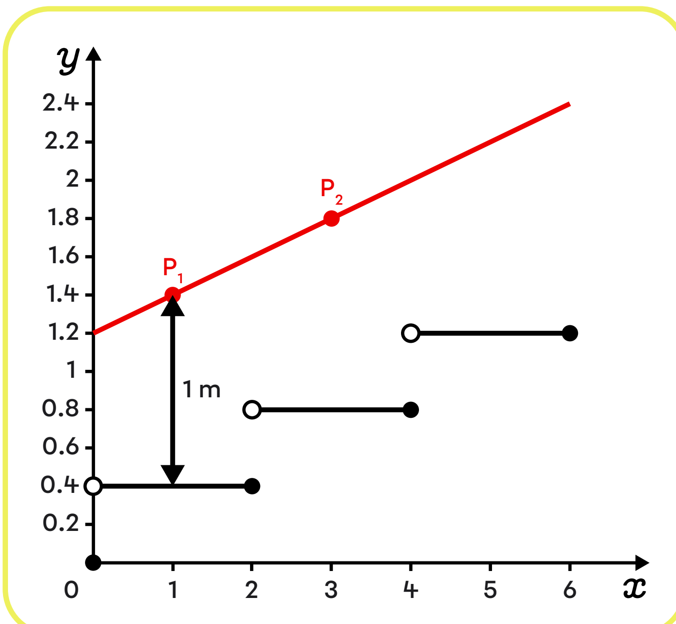
$$\begin{aligned} g(x) &= mx + k \\ g(x) &= \frac{1}{8}x + k \\ 1 &= \frac{1}{8}(2) + k \\ \frac{3}{4} &= k \end{aligned}$$

   Therefore, the rule is $g(x) = \frac{1}{8}x + \frac{3}{4}$.

2nd Case: Parameters a and b are negative

1. Choose the staircase rule (f(x)).

   Let's opt for an **ascending** staircase with **negative** parameters **a and b**.
   We'll use the following values:
   → If we select a = -0.4, each riser will be 0.4 m.
   → If b = -0.5, each step will have a length of $\frac{1}{0.5}$ = 2 m.
   → Because we selected negative a and b parameters, the staircase will be ascending, and each step will start with an empty point.
   The selected rule is therefore $f(x) = -0.4[-0.5x]$, and its graph is as follows:



2. Determine the rule for the handrail ($g(x)$).

   a) To determine the handrail rule, we need 2 points on the line.
   → Point P_1 is 1 m above the midpoint of the 1st step, which runs from 0 to 2 at a height of 0.4 m. The coordinates of P_1 are therefore (1, 1.4).
   → Point P_2 is 1 m above the midpoint of the 2nd step, which runs from 2 to 4 at a height of 0.8 m. The coordinates of P_2 are therefore (3, 1.8).

   b) Calculate the slope (m).

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1.8 - 1.4}{3 - 1} \\ &= 0.2 \end{aligned}$$

   c) Determine the y-intercept (k).

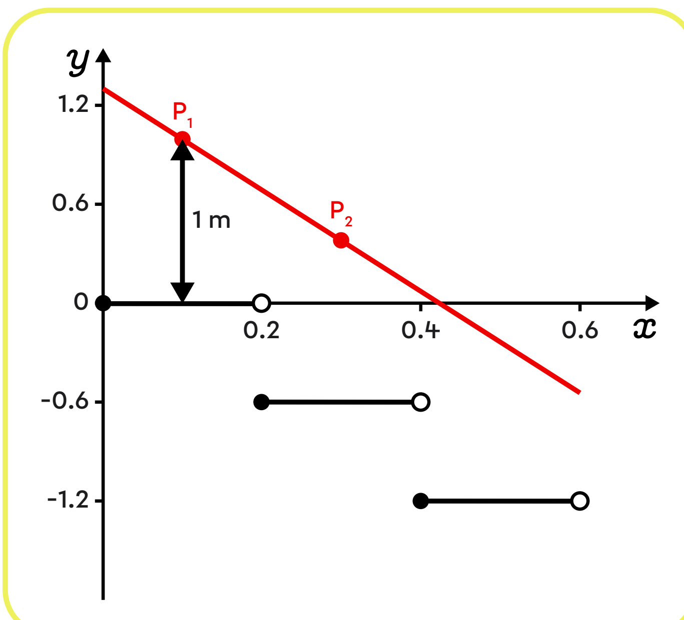
$$\begin{aligned} g(x) &= mx + k \\ g(x) &= 0.2x + k \\ 1.4 &= 0.2(1) + k \\ 1.2 &= k \end{aligned}$$

   Therefore, the rule is $\text{g}(\text{x}) = 0.2\text{x} + 1.2$.

3rd Case: Parameter a is negative and parameter b is positive

1. Choose the staircase rule ($f(x)$).

   Let's opt for a **descending** staircase with negative parameter a.
   We'll use the following values:
   → If we select a = -0.6, each riser will be 0.6 m.
   → If b = 5, each step will have a length of $\frac{1}{5}$ = 0.2 m.
   → Because a is negative and b is positive, the staircase will be descending, and each step will start with a full point.
   The selected rule is therefore f(x)=-0.6[5x], and its graph is as follows:



2. Determine the rule for the handrail (g(x)).

   a) To determine the handrail rule, we need 2 points on the line.
      → Point P_1 is 1 m above the midpoint of the 1st step, which runs from 0 to 0.2 at a height of 0 m. Therefore, the coordinates of P_1 are (0.1, 1).
      → Point P_2 is 1 m above the midpoint of the 2nd step, which runs from 0.2 to 0.4 at a height of -0.6 m. The coordinates of P_2 are therefore (0.3, 0.4).

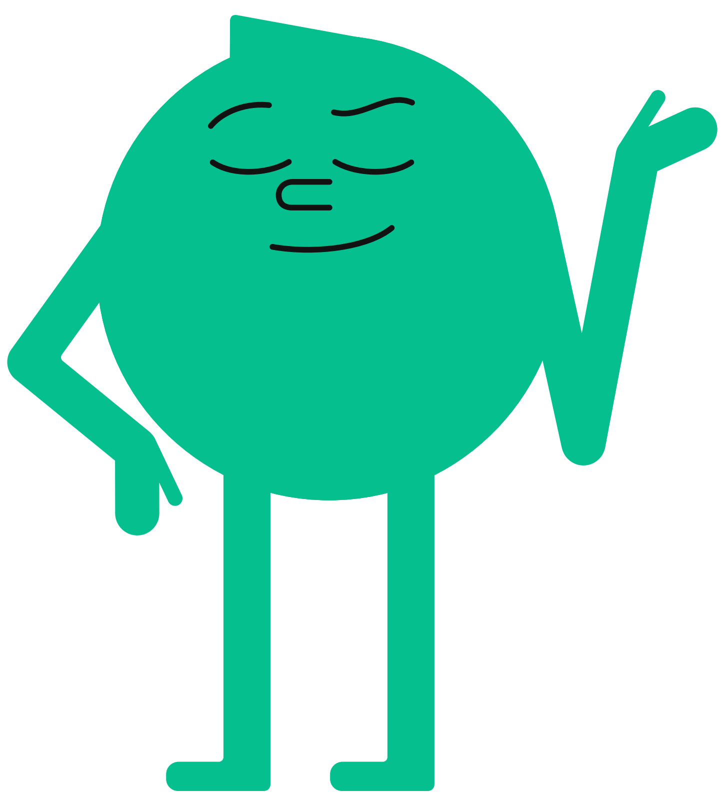
   b) Calculate the slope (m).

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0.4 - 1}{0.3 - 0.1} \\ &= -3 \end{aligned}$$

   c) Determine the y-intercept (k).

$$\begin{aligned} g(x) &= mx + k \\ g(x) &= -3x + k \\ 1 &= -3(0.1) + k \\ 1.3 &= k \end{aligned}$$

   Therefore, the rule is $g(x) = -3x + 1.3$.



Looking for Relationships

Tips
It's often best to work with fractions to see relationships more clearly. For example, it is easier to see that $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ than to see that $0.5 \times 0.25 = 0.125$.

1st case	$a = 0.5 = \frac{1}{2}$ $b = 0.25 = \frac{1}{4}$	$m = \frac{1}{8}$ $k = \frac{3}{4}$
2nd case	$a = -0.4 = -\frac{2}{5}$ $b = -0.5 = -\frac{1}{2}$	$m = 0.2 = \frac{1}{5}$ $k = 1.2 = \frac{6}{5}$
3rd case	$a = -0.6 = -\frac{3}{5}$ $b = 5$	$m = -3$ $k = 1.3 = \frac{13}{10}$

When we're looking for relationships, we often need to ask ourselves questions like the following:

- **For *m***
  - How can we get $\frac{1}{8}$ using $\frac{1}{2}$ and $\frac{1}{4}$?
  - How can we get $\frac{1}{5}$ using $-\frac{2}{5}$ and $-\frac{1}{2}$?
  - How can we get −3 using $-\frac{3}{5}$ and 5?
- **For *k***
  - How can we get $\frac{3}{4}$ using $\frac{1}{2}$ and $\frac{1}{4}$ (without forgetting the 1 m)?
  - How can we get 1.2 using −0.4 and −0.5 (without forgetting the 1 m)?
  - How can we get 1.3 using −0.6 and 5 (without forgetting the 1 m)?

Tips

- Try basic operations first
  (addition, subtraction, multiplication, and division).

Examples

$$\frac{1}{2}+\frac{1}{4}\stackrel{?}{=}\frac{1}{8} \qquad \frac{1}{2}-\frac{1}{4}\stackrel{?}{=}\frac{1}{8}$$

$$\frac{1}{2}\times\frac{1}{4}\stackrel{?}{=}\frac{1}{8} \qquad \frac{1}{2}\div\frac{1}{4}\stackrel{?}{=}\frac{1}{8}$$

Verify that **the relationship you've found** also works in the other 2 cases.

- Try to use only one parameter at a time.

Examples

	With parameter **b**	With parameter **a**
1st case	$1-b=1-\frac{1}{4}=\frac{3}{4}=k$	$1-\frac{1}{2}a=1-\frac{1}{2}\times\frac{1}{2}=\frac{3}{4}=k$
2nd case	$1-b=1-(-0.5)=1.5\neq k$	$1-\frac{1}{2}a=1-\frac{1}{2}\times -0.4=1.2=k$

- Analyze the graph and use your mathematical knowledge.

Examples

- The y-intercept of a linear function (**k**) is related to the *vertical scale*, as is parameter **a** of a greatest integer function.
- To determine parameter **k**, the *height* of the handrail from the midpoint of the step (**1 m**) must also be taken into account.

Relevant relationships:

- **For *m***
  In all cases, multiplying parameters a and b gives parameter m.
  - 1st case: $ab = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} = m$
  - 2nd case: $ab = -0.4 \times -0.5 = 0.2 = m$
  - 3rd case: $ab = -\frac{3}{5} \times 5 = -3 = m$

  We verified that it works equally well in all cases, i.e., with positive, negative, and opposing sign a and b parameters.

- **For *k***
  In all 3 cases, subtracting half the value of parameter a to the 1 m height gives the following:
  - 1st case: $1 - \frac{1}{2}a = 1 - \frac{1}{2}\left(\frac{1}{2}\right) = 1 - \frac{1}{4} = \frac{3}{4} = k$
  - 2nd case: $1 - \frac{1}{2}a = 1 - \frac{1}{2}(-0.4) = 1 - (-0.2) = 1.2 = k$
  - 3rd case: $1 - \frac{1}{2}a = 1 - \frac{1}{2}(-0.6) = 1 - (-0.3) = 1.3 = k$

We're now ready to formulate a complete answer.

Tips
- You must write your answer, i.e., your conjecture, using complete sentences. In other words, while " $m = a \times b$ " and " $k = 1 - \frac{1}{2}a$ " can be included in your conjecture, this is not the expected answer.
- It's better to respond with a relationship that doesn't work in every case than none at all.

Answer

- The slope of function g(x) is equal to the product of parameters a and b of function f(x), regardless of the sign of a and b.

  In algebraic form: $m = a \times b$

- The y-intercept k of function g(x) is obtained by subtracting half the value of parameter a to the 1 m height.

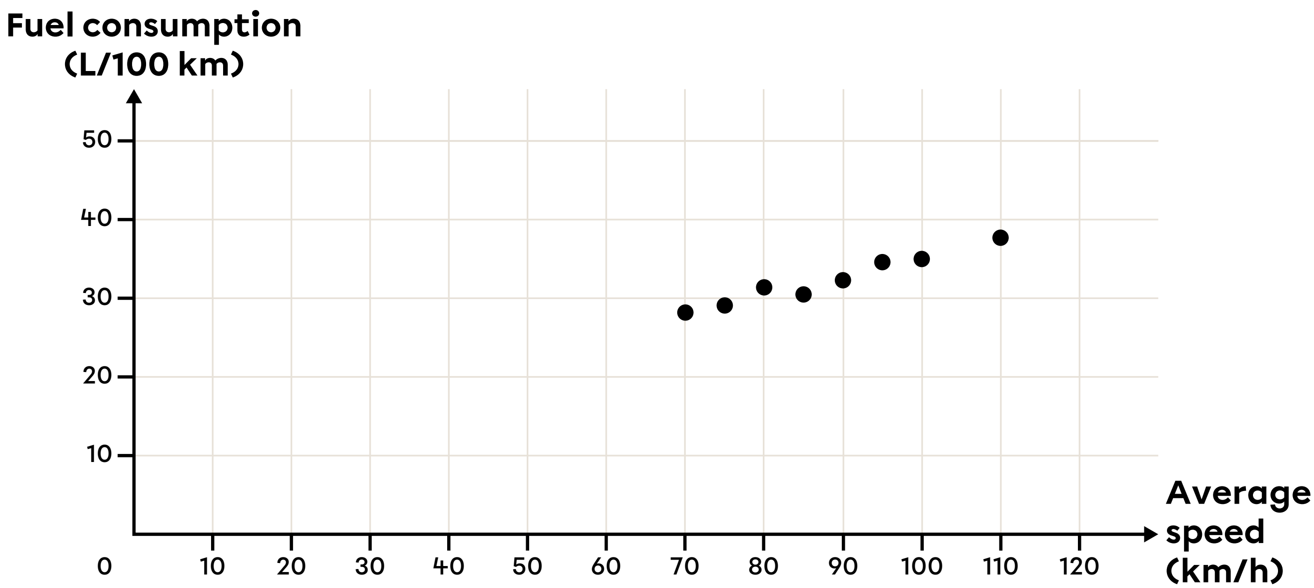
  In algebraic form: $k = 1 - \frac{1}{2}a$

Question 8

The main steps in solving the problem are as follows: We need to determine the regression line equation for the MetaCargo fleet data, then find the rule for fuel consumption as a function of average speed for the NovaFret fleet. Next, we need to use various speeds to analyze which truck fleet consumes less fuel.

A. Let's start with the **MetaCargo** fleet.

We create a scatter plot to see if there's a correlation between the variables.



There is a positive correlation between the variables, meaning the higher a truck's average speed, the higher its fuel consumption.

So, we're going to find the regression line rule. We can use either the **Mayer method** or median-median method. Let's use the Mayer method.

1. Order coordinates according to the independent variable.
This has already been done.

x: Average speed (km/h)	70	75	80	85	90	95	100	110
y: Fuel consumption (L/100 km)	28.2	29.1	31.4	30.5	32.3	34.6	35.0	37.7

2. Separate the distribution into 2 equal groups if possible.

	1st group				2nd group			
x: Average speed (km/h)	**70**	**75**	**80**	**85**	**90**	**95**	**100**	**110**
y: Fuel consumption (L/100 km)	28.2	29.1	31.4	30.5	32.3	34.6	35.0	37.7

3. Calculate the **average point** for each group (P_1 and P_2).

	1st group	2nd group
Average x-coordinate	$\overline{x}_1 = \dfrac{70+75+80+85}{4} = 77.5$	$\overline{x}_2 = \dfrac{90+95+100+110}{4} = 98.75$
Average y-coordinate	$\overline{y}_1 = \dfrac{28.2+29.1+31.4+30.5}{4} = 29.8$	$\overline{y}_2 = \dfrac{32.3+34.6+35.0+37.7}{4} = 34.9$
Average point	P_1 (77.5, 29.8)	P_2 (98.75, 34.9)

4. Find the rule of the regression line (y = ax + b) passing through points P_1 and P_2.

a) Calculate the slope.

$$\begin{aligned} a &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{34.9 - 29.8}{98.75 - 77.5} \\ &= \frac{5.1}{21.25} \\ &= 0.24 \end{aligned}$$

Therefore, $y = 0.24x + b$.

b) Calculate the value of b using point P_1 (77.5, 29.8).

$$\begin{aligned} y &= 0.24x + b \\ 29.8 &= 0.24(77.5) + b \\ 11.2 &= b \end{aligned}$$

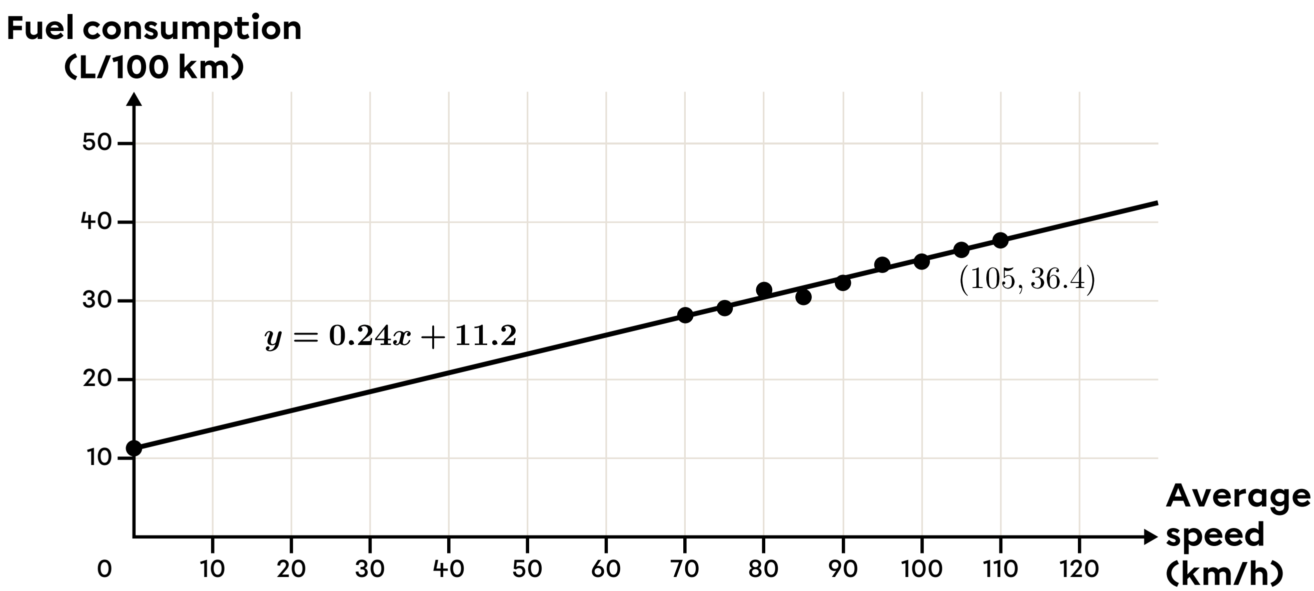
Thus, the regression line rule for MetaCargo is $y_1 = 0.24x + 11.2$.

Note: If you used the median-median method, you should get $y_1 = 0.236x + 11.18\overline{3}$.

5. Predict fuel consumption at 105 km/h using the line rule.

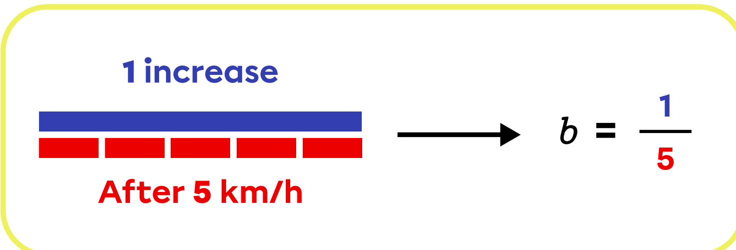
To predict fuel consumption for a truck driving at an average speed of 105 km/h, we need to replace x by 105 in the regression line.

$$\begin{aligned} y_1 &= 0.24x + 11.2 \\ &= 0.24(105) + 11.2 \\ &= 36.4 \text{ L/100 km} \end{aligned}$$



B. NovaFret.

1. Find the rule for the function that allows us to calculate fuel consumption for **NovaFret** trucks.
   - We have the model: $y = a(c)^{bx}$.
   - We have the y-intercept, which is the point (0, 12).
     The y-intercept corresponds to parameter a. Therefore, a = 12.
   - We know that fuel consumption increases by 6% every 5 km/h.
     An increase of 6% means that the base c equals 1.06,
     because 100% + 6% = 106% = 1.06.
     "Every 5 km/h" means that parameter b equals $\frac{1}{5}$.



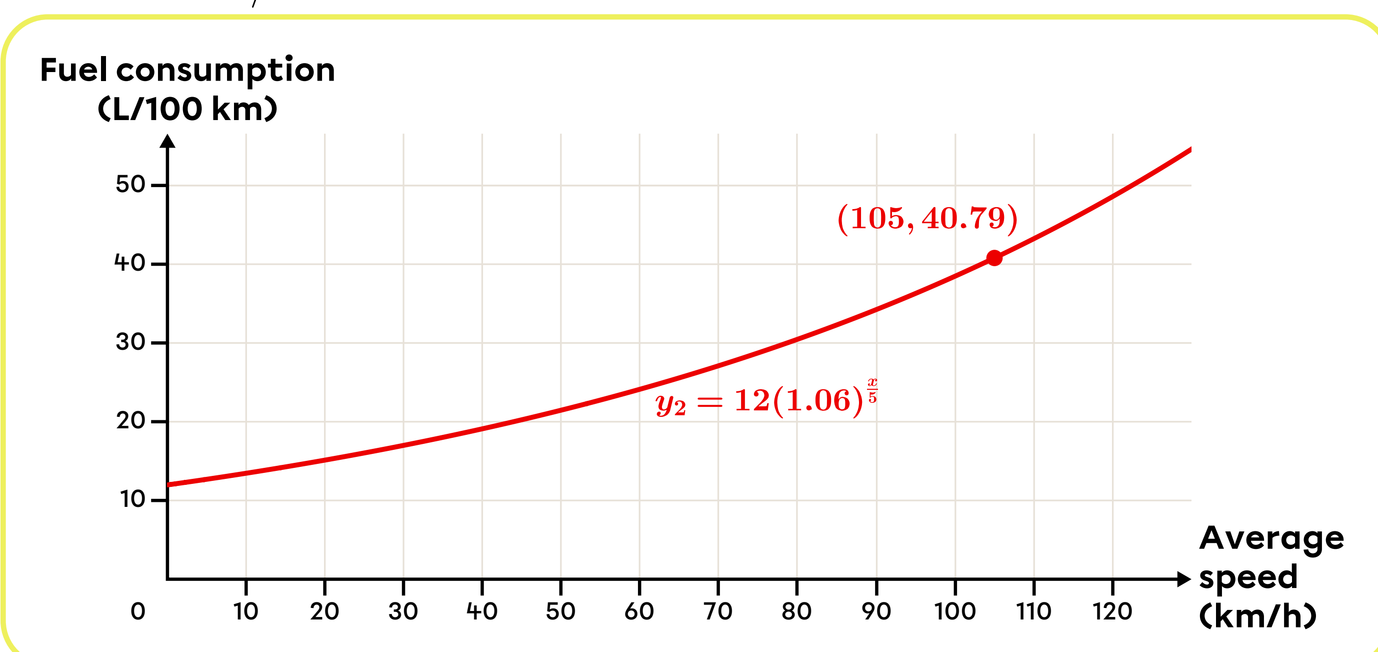
Therefore, the rule for the function that gives us fuel consumption relative to average speed (x) is as follows.

$$y_2 = 12(1.06)^{\frac{x}{5}}$$

2. Calculate fuel consumption at 105 km/h.

We replace x by 105 in the rule.

$$\begin{aligned} y_2 &= 12(1.06)^{\frac{x}{5}} \\ &= 12(1.06)^{\frac{105}{5}} \\ &\approx 40.79 \text{ L}/100 \text{ km} \end{aligned}$$



Answer: Given that the average route speed is 105 km/h, he should use trucks in the **MetaCargo** fleet.

Justification: According to the regression line, fuel consumption for the trucks in the MétaCargo fleet at 105 km/h is approximately 36.4 L/100 km, which is lower than fuel consumption for the trucks in the NovaFret fleet, which is around 40.79 L/100 km.